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RESEARCH OF BANKRUPTCY IN CASE OF GREAT PAYMENTS

ДОСЛІДЖЕННЯ БАНКРУТСТВА ЗА УМОВ ВЕЛИКИХ ВИПЛАТ

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Annotation. *We present the asymptotics of the probability of bankruptcy in the case of large payments distributed under subexponential laws, in particular in cases of distribution of Pareto (which is assigned by the distribution function* $F(x) = 1 - \left(\frac{k}{k+x}\right)^{\alpha}$, $\alpha > 1, k > 0, x > 0$*), Weibull distribution, and distribution Bektandera type I and type II. Also, the asymptotic relations for an optimal insurance rate in the case of the F-model and in terms of distribution distributed by the Weibull distribution, Pareto distribution, and log-normal distribution are presented.*

Key words: *the asymptotics of the probability of bankruptcy, "heavy tails", subexponential distributions, Pareto distribution, Weibull distribution, distribution Bektandera type I and type II, insurance rate, optimal insurance rate, F-model.*

1. Estimation of the probability of bankruptcy

Activity of insurance company characterized by different parameters, one of which is the probability of bankruptcy. We can say that the financial risk and the related risk of bankruptcy – the characteristics of each insurance company. Hence, an important task is to calculate the probability of bankruptcy and analysis of the results. Special interest, in our time, is determining the probability of bankruptcy for large payments, connected to natural disasters, terrorist acts, etc. Note also that large payment distributions described the so-called "heavy tails".

In the analysis of such distributions, including the distribution of Pareto, the question arose whether it is possible to obtain an estimate of probability of bankruptcy. A positive response was given to this question von Bahr [1] for the Pareto distribution and Thorin and Vikstad [2] for the log-normal distribution. Later there was a question whether there is a class of distributions with "heavy tails", which allows finding the probability of bankruptcy. The answer is given by Embrehts and Vereverbeke [3], which revealed a fundamental role of class subexponential distributions S in the theory of risk. This class include the log-normal distribution, distribution Pareto, distribution Barra, log gamma distribution, cut resistant distribution, Weibull distribution, distribution Bektandera type I and type II.[4, p. 195-196]

We consider the asymptotic behavior of the probability of bankruptcy in the case of "heavy tails". Note that the following statements are considered in [5], [6], [7].

Statement 1. [5] When payments have Pareto distribution, that is :

$$F(x) = 1 - \left(\frac{k}{k+x}\right)^{\alpha}, \alpha > 1, k > 0, x > 0,$$

then the asymptotic of bankruptcy probability $\varphi(u)$ is defined as:

$$\varphi(u) \sim \frac{\lambda \mathrm{k}^{\alpha}}{c(\alpha-1)-\lambda k}(k+u)^{-\alpha+1}, u \to \infty.$$

Statement 2. [6] Let payments distributed by Weibull distribution with a parameter $0 < \gamma < 1$ and the distribution function

$$F(x) = 1 - \exp\left(-c_1 x^{\gamma}\right), c_1 > 0, x > 0$$

then the asymptotic of probability of bankruptcy $\varphi(u)$ is given

$$\varphi(u) \sim \frac{\lambda}{\mathrm{c} \cdot \mathrm{c}_1^{\frac{1}{\gamma}} - \lambda \Gamma\left(1+\frac{1}{\lambda}\right)}\left[1 + \frac{\Gamma\left(\frac{1}{\gamma}; c_1 x^{\gamma}\right) - \Gamma\left(\frac{1}{\gamma}; 0\right)}{\gamma \cdot \Gamma\left(1+\frac{1}{\gamma}\right)}\right], u \to \infty.$$

Statement 3. [5] Let payments distributed Benktander type I:

$$1 - F(x) = \left(1 + \frac{2\beta \ln x}{\alpha}\right) x^{-(\alpha+1+\beta \ln x)} \; \alpha, \beta > 0, x > 1,$$

asymptotic of probability of bankruptcy $\varphi(u)$ given by the following equation:

$$\varphi(u) \sim \frac{\lambda(\alpha + 1 - \mathrm{u}^{-\alpha-\beta \ln x})}{c\alpha - \lambda(\alpha+1)}, u \to \infty.$$

Statement 4. [7] Let payments distributed Benktander type II:

$$1 - F(x) = \exp\left(\frac{\alpha}{\beta}\right) x^{-(1-\beta)} \exp\left\{-\frac{\alpha x^{\beta}}{\beta}\right\}, \alpha, \beta > 0, x > 1,$$

asymptotic of probability of bankruptcy $\varphi(u)$ given by the following equation:

$$\varphi(u) \sim \rho^{-1} \overline{F_I}(u) \sim \frac{\lambda}{(\alpha \mathrm{c} - \lambda(1+\alpha))} \exp\left(\frac{\alpha}{\beta}\right) \exp\left(-\frac{\alpha u^{\beta}}{\beta}\right), u \to \infty.$$

2. An optimal insurance rate in case of factorization model and great payments.

Let's assume that we are under conditions of factorization model, called F-Model [8, p.248].

Assume that number of insurance contracts N in the insurance portfolio is generally a random variable. We assign S_j, insurance sum, to each insurance contract with number *j*. Suppose Y_j - an insurance claim of contract *j*. It is obvious that $Y_j \le S_j$. For F-model, we call $X_j = \frac{Y_j}{S_j}$ - relative claim where the random variables X_j and S_j uncorrelated. It is obvious that insurance claim can be represented in the following way:

$$Y_j = X_j S_j \tag{1}$$

We will call the claims that satisfy the condition **(1)** factorizable [8].

For every contract the insurance premiums Z_j determined as follows:

$$Z_j = zS_j$$

where z – some constant for all insurance contracts (called the insurance premium rate or the insurance rate). It worth noting that the premiums in this model are random variables, that depend on S_j, which differs it from a classic problem.

We will take that all claims in the portfolio are factorizable, all random vectors (S_j, X_j) and N are jointly independent.

Sum of the premiums collected from insurance portfolio equals:

$$\overline{Z} = \sum_{j=1}^{N} Z_j$$

Sum of claims equals

$$\overline{Y} = \sum_{j=1}^{N} Y_j$$

If u_0 is a starting capital, then the final insurance fund is:

$$U = u_0 + \overline{Z} - \overline{Y} \tag{2}$$

The first problem related to **(2)** is the definition of asymptotic of random variable U distribution in case if z is known.

Second problem – is to define a minimal value for z that will provide us with acceptable results of insurance practice for the insurance portfolio.

We introduce the following limitations to define the insurance rate z:

$$z \geq EX_j \tag{3}$$

("average break even" condition)

$$P(U \geq 0) \geq Q \tag{4}$$

where Q – some predefined number $(0 < Q < 1)$, ("final non-bankruptcy" condition).

If z provides us with completion of conditions **(3)** and **(4)** then we call it admissible. z_0 – an optimal insurance rate, exact lower limit for z.

To simplify the records, we will say that random variable S is distributed the same as S_j and X the same as X_j. Suppose that S has got at least two finite moments. We define V as a coefficient of variation of random variable S:

$$V^2 = \frac{DS}{(ES)^2} = \frac{ES^2}{(ES)^2} - 1.$$

Define $H_j = S_j(z - I_j K_j)$, where random variables H_j are independent and jointly distributed. Then according to [8]

$$U = u_0 + \sum_{j=1}^{N} H_j$$

and for every u_0

$$P(U < x) = P(\sum_{j=1}^{N} H_j < x - u_0)$$

$\Phi(x)$ – a standard normal distribution function, $\Psi(x)$ - a reverse function to $\Phi(x)$. To simplify the problem, we will assume that $u_0 = 0$, N – constant value, Y_j satisfies (1).

Then $U = \sum_{j=1}^{N} H_j$. We suppose that N is big enough to approximate the U distribution by normal distribution law.

Suppose that the insurance portfolio claims are great. Payments with this size can be described by random variables with subexponential distributions. Then the following statements are fair.

Statement 5. [9] Assume that claim size is distributed by Weibull distribution with a parameter $0 < \gamma < 1$ and the distribution function

$$F(x) = 1 - \exp(-c_1 x^{\gamma}), c_1 > 0, x > 0$$

Then with these assumptions, for z_0 - optimal insurance rate the following correlation is fair:

$$z_0 \sim \frac{1}{c_1^{1/\gamma}} \cdot \Gamma\left[1 + \frac{1}{\gamma}\right] + \frac{\sqrt{\left(\frac{1}{c_1^{1/\gamma}}\right)^2 \cdot \left(\Gamma\left[1 + \frac{2}{\gamma}\right] - \left\{\Gamma\left[1 + \frac{1}{\gamma}\right]\right\}^2\right)}\left[1 + V^2\right]^{1/2} \Psi(Q)}{\left[N - V^2 \Psi^2(Q)\right]^{1/2}}$$

Statement 6. [10] Assume that claim size is distributed by Pareto with parameters $a > 0, \lambda > 0$ that is defined by distribution function:

$$F(x) = 1 - (\frac{\lambda}{\lambda + x})^a, x > 0.$$

Then with these assumptions, for z_0 - optimal insurance rate the following correlation is fair:

$$z_0 \sim \frac{\lambda}{a - 1} + \frac{\sqrt{\frac{a\lambda^2}{(a-1)^2 (a-2)}}[1 + V^2]^{1/2} \Psi(Q)}{[N - V^2 \Psi^2(Q)]^{1/2}}$$

Statement 7. [11] Assume that claim size is distributed by log-normal distribution with parameters $\mu \in \Re, \sigma > 0$ and distribution function:

$$F(x) = \frac{1}{2} + \frac{1}{2} erf\left[\frac{\ln x - \mu}{\sqrt{2\sigma^2}}\right]$$

Then with these assumptions, for z_0 - optimal insurance rate the following correlation is fair:

$$z_0 \sim e^{\mu + \sigma^2/2} + \frac{\sqrt{\left(e^{\sigma^2} - 1\right) e^{2\mu + \sigma^2}} [1 + V^2]^{1/2} \Psi(Q)}{[N - V^2 \Psi^2(Q)]^{1/2}}$$

It worth noting that with $V=0$ the results of statements 5, 6 and 7 come down to a classical case [8,p.238] that confirms accurateness of received results.

Conclusion

In this paper we considered the problem of bankruptcy probability definition in case of heavy tails and defined the asymptotic of bankruptcy probability in case of payments with Pareto, Weibull, Benktander type I and type II distributions. We have also presented the asymptotic relations for optimal insurance rates in case of the F-model and claims size distributed by the Weibull distribution, Pareto distribution, and log-normal distribution are presented.

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Аннотація. *Представлено асимптотики ймовірності банкрутства у випадку великих виплат розподілених за субекспоненційними законами, зокрема у випадках розподілу Парето* *(який задається* $F(x) = 1 - \left(\frac{k}{k+x}\right)^{\alpha}, \alpha > 1, k > 0, x > 0,$ *), розподілу Вейбулла, розподілів Бенктандера I та II типу. Також представлено асимптотичне співвідношення для оптимальної страхової ставки у випадку F-моделі та за умов виплат розподілених за розподілом Вейбулла, розподілом Парето та лог-нормальним розподілом.*

Ключові слова: *асимптотика ймовірності банкрутства, важкі хвости, субекспоненційні розподіли, розподіл Парето, розподіл Вейбулла, розподіл Бенктандера типу I та типу II, страхова ставка, F-модель.*

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