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CONSTRUCTING LINEARIZED EQUATIONS AND CALCULATING THEIR COEFFICIENTS ON THE EXAMPLE OF A GAS TURBINE ENGINE AS A PART OF ELECTRIC GENERATOR DRIVE

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Abstract. *The article discusses the peculiarities of using the ordinary least squares method for calculating the coefficients of linearized equations and is illustrated on the example of a gas turbine engine (GTE), used as a part of the drive of an electric generator of power plants of low and medium power. It has been shown that the use of the regularization method of normal equation system solution makes it possible to obtain adequate results in the systems with poorly conditioned matrices.*

Keywords: *gas turbine engine, linearized equations, linearization coefficients, Gaussian method, energy object.*

Introduction.

Recently, publications have been widely discussing the issues of improving quality indicators in the use of GTE in the industrial energy sector [1], [2].

GTE is used as a part of the electric generator drive of peak load of power plants, where the base load is provided by powerful industrial steam and gas turbines. In some cases, GTE manufacturers, on special order, design specialized options to use in stationary and transport power industry. Features of GTE design and their mathematical description, in particular, for this scheme can be found in the relevant references, for example [3].

Presentation of the main material.

We will consider an example of a GTE mathematical model, in which the equations are compiled taking into account the following assumptions: inertial properties of rotating masses of rotors only are taken into account; external air parameters are constant; loss of pressure in the air supply suction shaft does not depend on the GTE operation mode; the moment of resistance on the shaft of the power turbine depends only on the speed of its rotor.

This control object has one degree of freedom and the only control influence which is fuel consumption in the combustion chamber (B). The minimum number of differential equations of the first order corresponds to the number of energy batteries, in this case the mechanical energy of rotating masses. The system of linearized equations is as follows:

$$\begin{cases} T_1\delta\omega_1 = \delta M_{T2} - \delta M_{K1}, \\ T_2\delta\omega_2 = \delta M_{T1} - \delta M_{K2}, \\ T_3\delta\omega_3 = \delta M_{T3} - \delta M_{KC}. \end{cases} \quad (1)$$

The following notations are entered in the system (1):

$$\begin{aligned} &T_1 = \frac{\pi \cdot J_1 \cdot \omega_1}{30M_{10}}, M_{10} = M_{T20} = M_{K10}; \\ &T_2 = \frac{\pi \cdot J_2 \cdot \omega_2}{30M_{20}}, M_{20} = M_{T10} = M_{K20}; \\ &T_2 = \frac{\pi \cdot J_3 \cdot \omega_2}{30M_{10}}, M_{30} = M_{T30} = M_{C0}. \end{aligned} \quad (2)$$

where T_1, T_2, T_3 are time constants of the corresponding rotors; J_1, J_2, J_3 are the moments of rotor lag; M_{10} is a moment that develops in the output mode by a low-pressure turbine (LPT) ($M_{10} = M_{T20}$); or consumed by a low pressure compressor (LPC) ($M_{10} = M_{K10}$); M_{20} is a moment that develops in the output mode by a high-pressure turbine (HPT) ($M_{20} = M_{T10}$); or used by a high pressure compressor (HPC) ($M_{20} = M_{K20}$); M_{30} is a moment that develops in the output mode by the power turbine (PT) ($M_{30} = M_{T30}$); or consumed, which is equal to the moment of resistance ($M_{30} = M_{C0}$); ω_1, ω_2, ω_3 is the rotation frequency of LPC, HPC i PT correspondently.

In order to solve the value system (1) δM_{T2}, δM_{K2}, δM_{T1}, δM_{K2}, δM_{T3}, δM_C, that are in the right parts of the equations must be expressed through the parameters of the GTE cycle. The relationship between these parameters, taking into account the assumptions made above, is assumed to be algebraic. By applying the corresponding transformations, we can obtain that the right parts of the system equations (1) will have the same structure in composition of parameters ($\delta\omega_1$, $\delta\omega_2$, $\delta\omega_3$, δB). However, it is not advisable to make such a transformation for two reasons. Firstly, for a complete picture of the operation mode of the GTE; except of the dependencies $\delta\omega_1(t)$, $\delta\omega_2(t)$, $\delta\omega_3(t)$) under the specified law $\delta B(t)$, it is necessary to have parameters to determine the operation mode of compressors in relation to the pump line ($\delta\pi_{K1}$, $\delta\pi_{K2}$) and also know the change in gas temperature (δT_G) in front of a high-pressure turbine (π_{K1}, π_{K2} is a degree of air compression in LPC i HPC correspondently). Then, for the relative values of compressors and turbines, the following expressions can be written out:

$$\begin{cases} \delta M_{K1} = \delta M_{K1}(\delta\pi_{K1}, \delta\omega_1); \\ \delta M_{K2} = \delta M_{K2}(\delta\pi_{K1}, \delta\pi_{K2}, \delta\omega_1, \delta\omega_2); \\ \delta M_{T1} = \delta M_{T1}(\delta\pi_{K1}, \delta\pi_{K2}, \delta\omega_1, \delta\omega_2, \delta T_G); \\ \delta M_{T2} = \delta M_{T2}(\delta\pi_{K1}, \delta\pi_{K2}, \delta\omega_1, \delta\omega_2, \delta T_G); \\ \delta M_{T3} = \delta M_{T3}(\delta\pi_{K1}, \delta\pi_{K2}, \delta\omega_1, \delta\omega_2, \delta\omega_3, \delta T_G); \\ \delta M_C = \delta M_C(\delta\omega_3); \\ \delta\pi_{K1} = \delta\pi_{K1}(\delta\pi_{K2}, \delta\omega_1, \delta\omega_2, \delta\omega_3, \delta T_G); \\ \delta\pi_{K2} = \delta\pi_{K2}(\delta\pi_{K1}, \delta\omega_1, \delta\omega_2); \\ \delta T_G = \delta T_G(\delta\pi_{K1}, \delta\pi_{K2}, \delta\omega_1, \delta\omega_2, \delta B). \end{cases} \quad (3)$$

The formula structure according to the composition of parameters is derived from the mathematical description of physical phenomena occuring in GTE during its work in static and transitional modes.

The second reason why a certain number of algebraic equations should be preserved in addition to differential equations occurs in the features of the applied technique. The exclusion of algebraic equations will cause the right parts of the system equations (3) to become identical in composition of their parameters. Then the calculation of GTE in static mode ($\omega 1 = \omega 2 = \omega 3 = 0$) will not be enough to calculate the linearization coefficients – all equations will be built into one type

$$\alpha_1 \delta\omega_1 + \alpha_2 \delta\omega_2 + \alpha_3 \delta\omega_3 = \alpha_1 B,$$

where $\alpha 1$, $\alpha 2$, $\alpha 3$ are linearization coefficients, that are derived.

Thus, the first feature of the suggested method of calculating linearization coefficients according to the static characteristics of the control object has been established: linearization equations for which coefficients are determined must differ in the composition of parameters.

In case when the results are used, that contain the values $\omega_1 \neq 0, \omega_2 \neq 0,$ and $\omega_3 \neq 0$, then algebraic equations are not needed to determine the coefficients of linearization. The latter in the system of differential equations are located exactly to the multiplier at the derivative. However, as mentioned above, the physical meaning of the problem requires the conservation of parameters of πк1, πК2, TG to access the quality of the investigated processes. After the initial system of linearized equations is defined, a set of conditions is written for each equation, which are then given to the system of normal equations. Linearization coefficients are determined from the solution of normal equations. There is a standard approximation algorithm OLS in many computer programs for mathematical calculations, which can be used for this purpose. The range in which the parameters are sampled to make conditional equations is assigned purely tentatively, guided by an intuitive representation of the fairness of the linearization hypothesis.

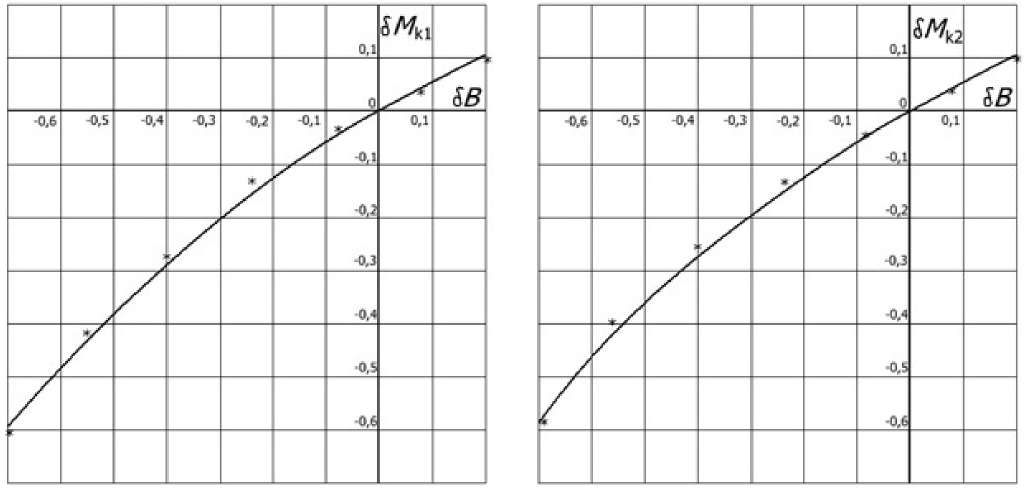
Table. 1 demonstrates the coefficients of linearized algebraic equations (2). Fig. 1−3 show the values calculated using linearization coefficients of the table and obtained from the thermal calculation of GTE characteristics according to nonlinear dependencies.

Table 1 - Coefficients of the linearized equations

Parameter in the left side of the equation	Linearization coefficients at parameters in the right side of the equation						
	$\delta\pi_{K1}$	$\delta\pi_{K2}$	$\delta\omega_1$	$\Delta\omega_2$	$\Delta\omega_3$	δT_G	δB
M_{K1}	0,6518	–	1,088	–	–	–	–
M_{K2}	0,3152	1,5382	0,2798	0,492	–	–	–
M_{T1}	0,220	0,9414	0,1961	0,4053	–	0,5303	–
M_{T2}	0,2648	0,8260	0,3552	0,6439	–	0,417	–
M_{T3}	0,3383	0,0083	0,1788	0,1772	0,3328	1,1395	–
M_C	–	–	–	–	1,6362	–	–
π_{K1}	–	0,2388	1,1889	0,0248	0,0243	0,0951	–
π_{K2}	0,1824	–	0,1713	0,2388	–	–	–
T_G	0,8925	0,7488	-0,992	-0,006	–	–	0,1139

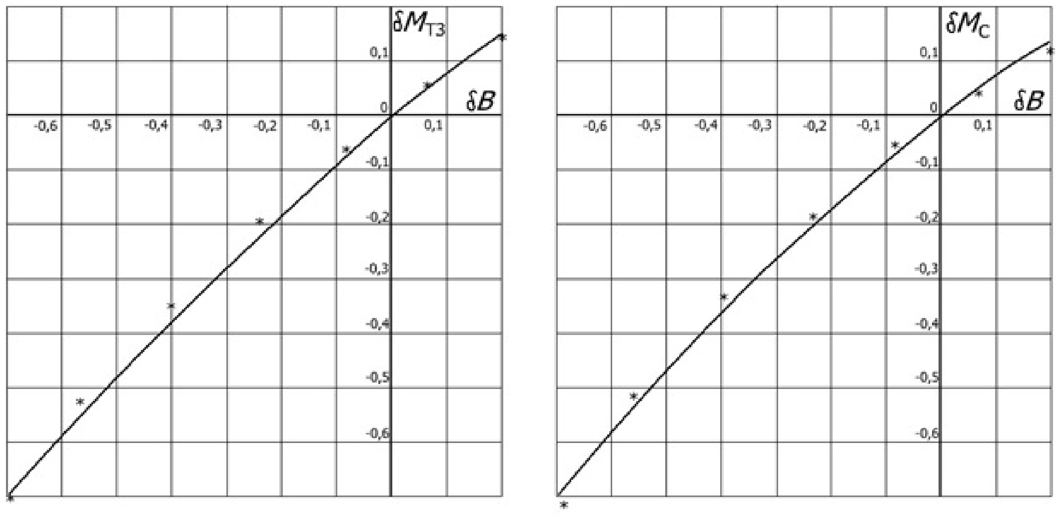
After analyzing fig. 1–3 it can be concluded that for the range of linearization adopted in the example of load linearization, the resulting system of linear equations approximates the transitional nonlinear dependence quite well.

The possible range by B for the use of the obtained linear equations is obtained from the graphs in Fig. 4−5, which demonstrates the appproximation error and marks the permissible error value. The greatest errors occur at appproximation of power turbine moments and load.



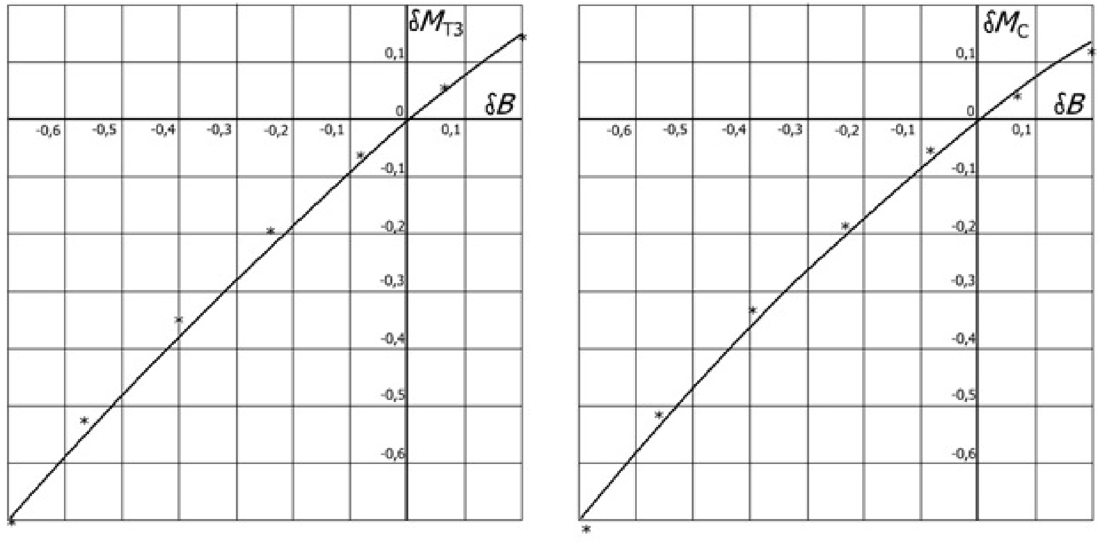
a) b)

Figure 1 - Dependency: ***a*) $\delta M_{K1} = \delta M_{K1}\delta(B)$, *b*) $\delta M_{K2} = \delta M_{K2}\delta(B)$; (curve line is an output nonlinear dependency; discrete points are values calculated according to lianearized equation)**



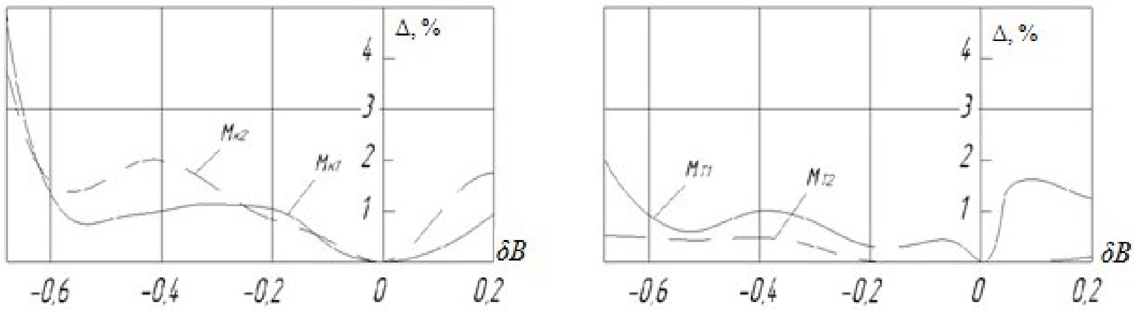
a) b)

Figure 2 - Dependency: a) $\delta M_{T1} = \delta M_{T1}\delta(B)$, b) $\delta M_{T2} = \delta M_{T2}\delta(B)$; (curve line is an output nonlinear dependency; discrete points are values calculated according to lianearized equation)



a) b)

Figure 3 - Dependency: a) $\delta M_{T3} = \delta M_{T3}\delta(B)$, b) $\delta M_C = \delta M_C\delta(B)$; (curve line is an output nonlinear dependency; discrete points are values calculated according to lianearized equation)



a) b)

Figure 4 - To the assessement of the acceptable linearization range for dependencies: *a*) δM_{K1}, *b*) δM_{K2}

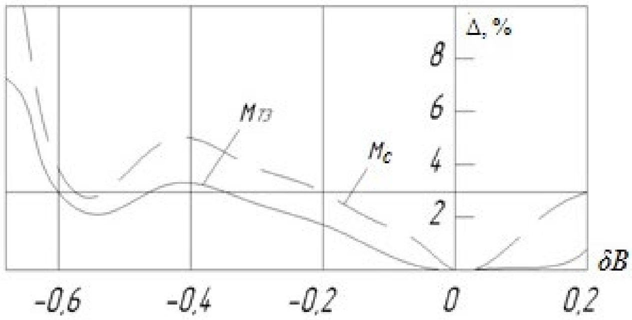


Figure 5 - To the assessement of the acceptable linearization range for dependencies: *a*) δM_{T3}, δM_C

When calculating the coefficients of linearization for approximation of the value δM_{T1}, the following system of normal equations was obtained:

$$\begin{cases} 0,3442x_1 + 0,1353x_2 + 0,2326x_3 + 0,2453x_4 + 0,1343x_5 = 0,4489; \\ 0,1353x_1 + 0,0533x_2 + 0,0913x_3 + 0,0968x_4 + 0,0527x_5 = 0,1769; \\ 0,2326x_1 + 0,0913x_2 + 0,1527x_3 + 0,1655x_4 + 0,0908x_5 = 0,3029; \\ 0,2453x_1 + 0,0968x_2 + 0,1655x_3 + 0,1759x_4 + 0,0955x_5 = 0,3213; \\ 0,1343x_1 + 0,0527x_2 + 0,0908x_3 + 0,0955x_4 + 0,0525x_5 = 0,1749. \end{cases}$$

Solving the Gaussian equation system with the allocation of the main elements on a personal computer gives the following values for unknown values

$$x_1 = 86,5839; x_2 = -19,7215; x_3 = -155,0071; x_4 = 1,0259; x_5 = 68,0311.$$

When we substitute the received solutions, for example, in the first of the equations of the system we get a fairly good convergence

$$0,3442 \cdot 86,5839 - 0,1353 \cdot 19,7215 - 0,2326 \cdot 155,0071 + 0,2453 \cdot 1,0259 +$$

$$+0,1343 \cdot 68,0311 = 0,3675(0,4489).$$

When calculating the system of equations by the Gaussian method with the selection of the main elements, there are a number of signs that immediately indicate the poor condition of the system. These features include: the appearance of a "small" conduction element, a "large" numerical solution, a "large" residual vector [4]. In this example, the manifestation of poor condition of the source system is to obtain "large" numerical values. Since this problem has quite a certain physical content, these "large" solutions are not consistent with a priori ideas about the nature of the object. For the adjusting multiplier μ = 0,99, entered on recommendations [4], the solution of the system will be

$$x_1 = 0,6217; x_2 = 0,0129; x_3 = 1,253; x_4 = -0,3559; x_5 = 0,2318.$$

Substitution of the obtained values in the first equation of the source system gives

$$0,3442 \cdot 0,6217 - 0,1353 \cdot 0,0129 - 0,2326 \cdot 0,2326 -$$

$$-0,2453 \cdot 0,3559 + 0,1343 \cdot 0,2318 = 0,4508(0,4489).$$

For the regulatory multiplier μ = 1,01 we get

$$x_1 = 0,22; x_2 = 0,9414; x_3 = 0,1961; x_4 = 0,5903; x_5 = 0,4053.$$

A similar check is as follows

$$0,3442 \cdot 0,22 + 0,1353 \cdot 0,9414 + 0,2326 \cdot 0,1961 -$$

$$-0,2453 \cdot 0,5903 + 0,1343 \cdot 0,4053 = 0,4479(0,4489).$$

As a reliable solution, the values $x_1 ... x_5$ obtained for μ = 1,01 are taken, since the absence of negative coefficients inevitably follows from the analysis of the physical essence of the identified object. The effectiveness of the applied regulatory method is clearly visible from fig. 5, which demonstrates an appproximation error.

Summary.

The construction of a linear model of an energy object based on OLS was discussed above in detail. The use of the regularization method of normal equation solution allows to get reliable results in the systems with poorly conditioned matrices. The essence of the results can be reduced to the following.

If the calculation results of energy object static characteristics are used to obtain coefficients of linear equations, then linear equations can be used:

1) for static calculations with small changes in individual parameters;

2) to obtain transient processes in a small range;

3) to obtain transient processes in the entire permissible linearization range in case of relatively small inconsistency of the parameters of the energy object (slow dynamics).

If the results of calculations or tests containing derivatives of parameters that characterize the parameters of the energy object are used to obtain coefficients of linear equations, then linear equations will be suitable for describing transition modes with a single restriction on the permissible range of linearization.

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