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# UDC 62.95:681.5 MATHEMATICAL MODELING OF THE LOW-TEMPERATURE SEPARATION PROCESS

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Annotation. The gas extracted from the well contains impurities - oxygen, nitrogen, hydrogen sulfide, as well as condensate, heavy hydrocarbons, butane, propane and their derivatives. Before the gas is sent to consumers, it is cleaned. The gas purification process proceeds in two stages; at the first of them, mechanical impurities are removed, and at the second stage, water, heavy carbohydrates and condensate are removed from the gas.

In the case when gas under high pressure (not less than 14 MPa) comes out of the well, an effective method of its purification is low-temperature separation.

A condition for the effectiveness of the low-temperature separation process is compliance with the technological regulations, which are provided by means of local automation, which performs the functions of stabilizing the parameters of the technological process. Existing automatic stabilization systems are single-circuit and do not take into account the presence of cross- connections between technological parameters, which significantly reduces the efficiency of the low-temperature separation process.

Therefore, the goal of the work was to create a mathematical model of the low-temperature separation process, which made it possible to identify objectively existing relationships between technological parameters, which is the methodological basis for the synthesis of an effective control system for the separation process.

of the low-temperature separation process in terms of "input-output" were obtained on the basis of the law of conservation of the amount of matter and the laws of exchange of substances between the liquid and gas phases. The linearization of the mathematical model of the dynamics of the low-temperature separation process was performed, and on its basis, the matrix transfer function of the object was obtained, which made it possible to formalize the channels of internal interactions between input and output values.

*Key words:* low-temperature separation, mathematical model, linearization, transfer functions, structural diagram.

## Introduction

The gas extracted from the borehole, contains mechanical particles, water, as well as accompanying substances - oxygen, nitrogen, hydrogen sulfide, as well as condensate, heavy hydrocarbons, butane, propane and their derivatives. Therefore, the gas is pre-cleaned before it is sent to consumers. Gas purification takes place in two stages. In the first stage, the gas is cleaned of mechanical impurities, and in the second stage, water, condensate and heavy carbohydrates are removed from the gas. In the case when the extracted gas comes under high pressure (at least 14 MPa), low-temperature separation (LTS) is used for gas purification.

The effectiveness of the NTS process depends on the proper support of the technological regulation, which is defined for the low-temperature separator. As a result of the change in the conditions of the NTS process, there is a deviation of the mode parameters from those defined by the technological regulations. Therefore, to maintain the technological parameters at the set values, automatic stabilization systems are used, which are mostly single-circuit and do not take into account the presence of internal connections between the technological parameters, which impairs the efficiency of the separation process, and also leads to the loss of such valuable components as condensate and heavy hydrocarbons.

## Analysis of literary sources

A feature of the NTS process is the change in the state of the gas flow as it passes through technological devices, which complicates its mathematical description.

The gas entering the NTS installation is throttled, which causes a turbulent flow of gas, and after entering the separator, the flow changes to a uniform state [1]. Depending on the nature of the flow - subcritical or supercritical, flowing through the throttle device, the gas consumption is calculated according to various formulas [2]. When passing gas under high pressure through the throttle, the Joule-Thomson effect occurs, which causes a decrease in temperature and pressure. When the pressure decreases by 1 MPa, the gas temperature decreases [3] by 5.5 °C.

Mathematical modeling of the low-temperature separation process itself is based on the use of regularities that occur in the vapor-liquid system. If the vapor-liquid system is in an equilibrium state, then there is a process of mass exchange between the liquid and gas phases. The driving force of such a process is the chemical potential. For the first time, the concept of chemical potential was introduced by Gibbs during the study of equilibrium conditions between the liquid and gas phases [4]. The equilibrium state of the "liquid-gas" system is characterized by a certain temperature T and pressure P. Let's assume that the "liquid-gas" system, which is in equilibrium, consists of *i* components. Then the condition of thermodynamic equilibrium is given by the following equality [5]:

$$f_i^{(V)} = f_i^{(L)},$$

where f - fugitiveness (volatility); V - liquid; L - steam

When modeling the process of low-temperature separation, one of the main tasks is to identify the relationship between the composition of the mixture and the fugitives  $f_i^{(V)}$  and  $f_i^{(L)}$ .

Fugacity is a measure of the deviation of the real liquid-gas system from the ideal (ideal gas, ideal solution) and determines the pressure of the real system that exerts the same effect on the environment as the ideal system. As the real state of the gas (real liquid) approaches the ideal  $f \rightarrow P$ . It follows from the last condition that fugacity is measured in units of pressure. The degree of deviation of the properties of



the real "liquid-gas" system from the ideal is characterized by the fugacity coefficient  $\gamma f/P$ . If the composition of the mixture is expressed in mole fractions *Y* (dimensionless units), then  $\gamma = \frac{1}{V}$ . For a mixture of ideal gases  $\gamma_i f_i/P_i = 1$ .

The fugacity i-of the component depends on the temperature, pressure and composition of the mixture. It should be noted that there is no analytical dependence for determining the fugacity as a function of the listed parameters. Therefore, the determination of fugacity is a rather difficult task, which is based on experimental studies.

For a gas system, the following relationship [5] holds:

 $PY_i = X_i f_i(T) \gamma_i,$ 

where  $Y_i$ ,  $X_i$ - mole fractions of gas and liquid *i*-of the component in the mixture.

The value  $PY_i$  is called partial pressure.

The vast majority of mathematical models of the NTS process are oriented towards solving technological problems. Mathematical models in terms of "inputoutput" are necessary for the synthesis of automatic control systems. Models of this type partially obtained in works [1] and [6].

Single-circuit systems of automatic regulation of pressure, liquid level and temperature are used to stabilize the technological parameters of the NTS [7].

Thus, the mathematical description of the low-temperature separation process is mostly focused on the synthesis of single-loop control systems [8]. As the analysis of literary sources shows, there are internal connections between the input values in the separators of both the first and second stages of separation of the gas mixture extracted from gas boreholes. Failure to take into account internal connections deteriorates the quality of the management process.

*The purpose of the work.* The purpose of the work is to increase the efficiency of the low-temperature separation process by creating mathematical models and synthesizing automatic control systems, taking into account the internal cross-connections that exist between the technological parameters of the process.

The set goal required solving the following tasks in the work:

- on the basis of physical regularities inherent in the process of low-temperature separation, create its static model;
- identify the input and output values of the control object, form basic assumptions and create a mathematical model of the low-temperature separation process in "input-output" terms;
- linearize the mathematical model of the dynamics of the low-temperature separation process and determine the matrix transfer functions and their properties;
- synthesize a system of automatic control of the low-temperature separation process taking into account the internal relationships between technological parameters and choose a method of calculating the parameters of the regulator setting, as well as evaluate the quality of the control process.

# Mathematical model of the low-temperature separation process

The schematic functional scheme of the NTS installation is shown [6] in Fig. 1.

The gas extracted from the borehole goes to the complex gas preparation unit (UCPG). After UKGP, natural gas is pre-throttled (or without throttling) and fed to the inlet of separator C-1 (Fig. 1). In the C-1 separator, the droplet liquid is separated. Further, the gas through the T1 heat exchanger enters the C-2 separator through a fitting or an ejector. In the T1 heat exchanger, the gas is cooled with the help of a coolant (gas), which is fed into the intertube space of the T1 heat exchanger. From the outlet of the C-2 separator, the dried gas enters the heat exchanger, where it is heated and, after measurement, is fed to the gas collector. To prevent water formation diethylene glycol (DEG) is added to the gas inlet in front of the C-2 separator. Due to the difference in the densities of the condensate and DEG, the liquid phases are gravitationally separated and the separated DEG falls into containers P\_1 and P-2 (Fig. 1).



Figure. 1. Simplified functional diagram of the NTS installation

The mathematical model of the NTS process was developed under the following assumptions [5]:

- the temperature regime in the C-2 separator (Fig. 1) is maintained unchanged;
- the density of the liquid phase in the separator is constant;
- the known chemical composition of the gas entering the separation and mole fraction of  $\mu_{0i,s}$  its components;
- the mole fraction of gas in the mixture at the entrance to the C-2 separator is known and remains unchanged during the observation period;
- gas and condensate obey the laws of ideal solutions.

Based on the analysis of the low-temperature separation process as an object of automatic control, we will adopt the following variable models:

• *initial values* – gas pressure P and liquid phase level H in the separator;

• *input values* – degree of opening  $U_1$  and  $U_2$  executive bodies 1 and 2, which are installed at the outlet and inlet of separator C-2 (Fig. 1);



• disturbances – pressures in the pipelines  $P_1$  (at the inlet to the C-2 separator),  $P_2$  (at the outlet from the C-2 separator) and  $P_3$  (at the liquid drain from the C-2 separator, Fig. 1).

In the conditions of the equilibrium state, the low-temperature separation proceeds as an exchange between the liquid and gas phases. The driving force of such a process is the chemical potential. Quantification of chemical potential is called volatility. This estimate was introduced by Gypsum to characterize general equilibrium conditions.

Volatility is some function of pressure and shows that a real system exerts the same effect on the environment as an ideal system (ideal gas, ideal solution). Volatility is a measure of the deviation of an ideal system from a real one under constant external conditions and is measured in units of pressure.

Size

$$\chi_i = \frac{f_{i,s}}{\mu_{i,s} f_{0i,s}} \tag{1}$$

is called the activity coefficient, a

$$\varphi_i = \frac{f_{i,g}}{\mu_{i,g} f_{0i,g}} \tag{2}$$

called the volatility coefficient.

In formulas (1), the following designations are adopted:  $\mu_{i,s}$  - mole fraction *i*-of the th component in the solution;  $f_{0i,s}$ - volatility *i*-of the pure component;  $f_{i,s}$  volatility *i*-of the component in the liquid mixture at constant atmospheric pressure. The physical quantities in formula (2), which are assigned the index " g ", have the same meaning as in formula (1).

In the case when the vapor-gas system is in equilibrium, there is equality  $f_{i,s} = f_{i,g}$ . If the values and are  $f_{i,g}$  determined from formulas (1) and (2)  $f_{i,s}$ , then the last equality will be as follows:

$$\chi_{i}\mu_{i,s}f_{0i,s} = \varphi_{i}\mu_{i,g}f_{0i,g}.$$
 (3)

It is assumed that the pure component is in ideal gas conditions. In this case  $f_{0i,g} = P$ . Taking into account the last assumption, the relation (3.4) will be as follows:

$$\chi_i \mu_{i,\mathrm{s}} f_{0i,\mathrm{s}} = \varphi_i \mu_{i,\mathrm{g}} P \, .$$

From the last equation we find

$$\mu_{i,g}P = \frac{\chi_i \mu_{i,s} f_{0i,s}}{\varphi_i}.$$
(4)

The value  $\mu_{i,g}P$  is called partial pressure.

and  $f_{0i,s} = P_{pp}^{(0)}$  an ideal solution is used to describe the equilibrium state of a vaporliquid system, then the following  $f_{i,g} = P$  relations  $P_{pp}^{(0)}$  hold P: pairs over this component.

Under such conditions, Dalton's law for the gas phase holds

$$P_i = \mu_{i,g} P$$

and Raoult's law for the liquid phase

$$P_i = \mu_{i,s} P_{pp}^{(0)},$$

where  $P_i$  is the partial pressure;  $P = \sum_{i=1}^{n} P_i$  n – the number of components that make up the vapor gas system.

When the steam-gas system is in an equilibrium state, then the following relationship holds:

$$\mu_{i,g} P = \mu_{i,s} P_{pp}^{(0)} \,. \tag{5}$$

If we introduce a function  $f_i(T) = \frac{f_{0i,s}}{\varphi_i}$  that determines the relationship between the pressure and the boiling temperature of the pure *i*-component, then formula (4) will be as follows:

$$\mu_{i,g}P = \chi_i \mu_{i,s} f_i(T).$$
(6)

It follows from the comparison of formulas (5) and (6) that for the equilibrium state of the vapor-gas mixture, the following relationship holds:  $f_i(T) = P_{pp}^{(0)}$ .

For non-ideal systems, relation (6) is not fulfilled due to the influence of the mixture components on the activity coefficient of each component. This means that

$$\chi_i = \chi_i \left( \mu_{1,s}, \mu_{2,s}, \ldots, \mu_{n,s} \right).$$

Thus, in the case of an ideal vapor-gas system, its state is described by formula (5), and for a non-ideal vapor-gas system, it is necessary to take into account that the activity coefficient  $\chi_i$  is not a constant value, but a function of the mole fraction *i*-of those components in the system.

Let's assume that the inlet of the C-2 separator (Fig. 1) receives a gas mixture consisting of M moles. Denote by  $M_{con}$  and  $M_g$  the number of moles of condensate and gas in the total flow of the gas mixture. Relation

$$f_d = \frac{M_{con}}{M} \tag{7}$$

determines the mole fraction of condensate, and

$$1 - f_d = \frac{M_g}{M} \tag{8}$$

the total part of the gas in the mixture that entered the C-2 separator.

Knowing mole fractions  $\mu_{i,s}$  and  $\mu_{i,g}$  condensate and liquid phase *i* – of those components in the gas mixture, we will formulate the material balance equation for *i* – the component

$$\mathbf{M}\boldsymbol{\mu}_{0i,s} = \mathbf{M}_{con}\boldsymbol{\mu}_{i,s} + \mathbf{M}_{g}\boldsymbol{\mu}_{i,g}.$$

If we now take into account (3.8) and (3.9), the last equation will be as follows:

$$\mu_{0i,s} = f_d \mu_{i,s} + (1 - f_d) \mu_{i,g}.$$
(9)

For an ideal vapor-liquid system between  $\mu_{i,s}$  and  $\mu_{i,g}$  there is the following relationship [4]:

$$P\mu_{i,g} = \mu_{i,s}f_i(T)$$

We determine from the last formula

$$\mu_{i,g} = \frac{\mu_{i,s}f_i(T)}{P}$$

and substitute the obtained value  $\mu_{i,g}$  in (3.10). Then

$$\mu_{i,s} = \frac{\mu_{0i,s}}{f_d + (1 - f_d) f_i(T) P^{-1}}.$$

Since the assumption is made that the temperature regime in the C-2 separator (Fig. 1) is maintained constant, then  $f_i(T) = f_i = \text{const.}$  Ago

$$\mu_{i,s} = \frac{\mu_{0i,s}}{f_d + (1 - f_d) f_i P^{-1}}.$$
(10)

The known value  $\mu_{i,s}$  allows you to determine the number of moles i – th component of the condensate in the liquid phase

$$\mathbf{M}_{con,i} = \mathbf{M}_{con}\boldsymbol{\mu}_{i,s}$$

In accordance with formula (3.8)  $M_{con} = Mf_d$ . Therefore  $M_{con,i} = Mf_d\mu_{i,s}$ . In the last expression, we  $\mu_{i,s}$  substitute its value from (10). As a result, we get

$$M_{con,i} = M f_d \frac{\mu_{0i,s}}{f_d + (1 - f_d) f_i P^{-1}}.$$
 (11)

If the condensate contains  $N_{con}$  components, then at the exit from the C-2 separator (Fig. 1), we will have the following total number of moles of condensate

$$\mathbf{M}_{con}^{(out)} = \sum_{i=1}^{N_{con}} \mathbf{M}_{con,i} \; .$$

After taking into account the value  $M_{con,i}$  determined by formula (11), we obtain

$$\mathbf{M}_{con}^{(out)} = \mathbf{M} f_d \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s}}{f_d + (1 - f_d) f_i P^{-1}}.$$
 (12)

It is obvious that the number of moles of gas at the outlet of separator C-2 is the difference between the total number of moles of the gas mixture and the number of moles of condensate, i.e.

$$\mathbf{M}_{g}^{(out)} = \mathbf{M} - \mathbf{M}_{con}^{(out)}.$$

Since the value  $M_{con}^{(out)}$  is calculated by formula (12), then

$$\mathbf{M}_{g}^{(out)} = \mathbf{M} \left( 1 - f_{d} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s}}{f_{d} + (1 - f_{d}) f_{i} P^{-1}} \right)$$

Denote by the  $\eta_i$  weight concentration of each component of the mixture entering the C-2 separator. Let the  $\mu_{w,i}$  molar mass of *i* – the th component of the mixture. Then the mass of *i* – the th component in the mixture will be as follows:

$$G_i = \eta_i G_c, \tag{13}$$

where  $G_c$ -the total mass of the mixture entering the C-2 separator (Fig. 1), and the

total number of moles i – of the component in the mixture can be calculated as follows:

$$\mathbf{M}_i = \frac{G_i}{\mu_{w,i}} \,. \tag{14}$$

If  $N_c$  the number of components in the mixture entering the C-2 separator, then the total number of moles will be as follows:

$$\mathbf{M} = \sum_{i=1}^{N_c} \mathbf{M}_i \; .$$

Taking into account what is  $M_i$  calculated by formula (14), we will have

The value  $M_i$  can be found using formula (14). Then the last ratio will be:

$$\mathbf{M} = \sum_{i=1}^{N_c} \frac{G_i}{\mu_{w,i}}.$$

Since  $G_i$  can be found by formula (3.14), then

$$M = G_c \sum_{i=1}^{N_c} \frac{\eta_i}{\mu_{w,i}}.$$
 (15)

By analogy with formula (12), we can claim that

 $M_{con}^{(in)} = M f_d \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s}}{f_d + (1 - f_d) f_i P^{-1}}, \text{ but in this case the total number of moles should}$ be calculated according to (3.16). So,

$$\mathbf{M}_{con}^{(in)} = G_c f_d \sum_{j=1}^{N_c} \frac{\eta_j}{\mu_{w,j}} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s}}{f_d + (1 - f_d) f_i P^{-1}} \,.$$

Now we will find the amount of condensate entering separator 4 (Fig. 3.1)

$$G_{w}^{(in)} = G_{c} f_{d} \sum_{j=1}^{N_{c}} \frac{\eta_{j}}{\mu_{w,j}} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s} \mu_{w,i}}{f_{d} + (1 - f_{d}) f_{i} P^{-1}}.$$
 (16)

Let's move from the total amount of condensate to mass consumption. For this purpose, we introduce the following designations:  $m_{con}$ - the mass of condensate entering the C-2 separator per unit of time;  $m_{mix}$ - mass flow rate of the mixture at the inlet of the C-2 separator. Then, based on formula (16), we can write

$$m_{con} = m_{mix} f_d \sum_{j=1}^{N_c} \frac{\eta_j}{\mu_{w,j}} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s} \mu_{w,i}}{f_d + (1 - f_d) f_i P^{-1}}.$$
 (17)

the following relationship exists  $m_{mix} = m_{con} + m_g$  between the mass flow rates of the mixture  $m_{mix}$  and gas:  $m_g$ , then  $m_g = m_{mix} - m_{con}$ , or, taking into account (3.18), we have

$$m_g = m_{mix} \left( 1 - f_d \right) \sum_{j=1}^{N_c} \frac{\eta_j}{\mu_{w,j}} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s} \mu_{w,i}}{f_d + (1 - f_d) f_i P^{-1}}.$$
 (18)

There is a certain amount of water in the gas mixture that enters the C-2 unit for separation (Fig. 1). Its maximum amount is characterized by moisture content. This indicator depends on the dew point of the mixture. The dew point is determined by the temperature at which the gas becomes saturated at a given pressure. If the system (gas mixture) is in an equilibrium state, then the moisture content of the gas is calculated according to the Bukachek's formula



$$b_{mc} = a_{m,1} / P + a_{m,2},$$

where  $a_{m,1}$  and  $a_{m,2}$  are empirical coefficients.

mass of water that entered the separator can be determined by the known value of the moisture content  $b_{mc}$ 

$$m_w = b_{mc} m_{mix}^{(v)},$$

where  $m_{mix}^{(v)}$  is the volume flow of water in the initial mixture. Since it is massive  $m_{mix}$  and voluminous  $m_{mix}^{(v)}$  costs are related by the ratio  $m_{mix}^{(v)} = m_{mix} / \rho_{mix}$ , where  $\rho_{mix}$  is the density of the initial gas mixture, then

$$m_w = b_{mc,1} m_{mix} / \rho_{mix}. \tag{19}$$

The density  $\rho_{mix}$  is calculated according to the following formula:

$$\rho_{mix} = \frac{1}{N_A} \sum_{i=1}^{N_c} \mu_{0i,s} \mu_{w,i} ,$$

where  $N_A$  is Avogadro's number.

To prevent water formation, glycol (Fig. 1) is supplied to separator 4 in quantity

$$m_{gl} = \frac{\Delta m_w z_2}{z_1 - z_2},\tag{20}$$

where  $z_1$ , are  $z_2$  the relative units of the amount of glycol at the inlet of the heat exchanger T1 and at the outlet of the separator C-2 (Fig. 1).

Under the influence of glycol in the gas flow at the exit from separator 4, the amount of moisture changes by the following amount:

$$\Delta m_{w}=m_{w}-m_{w}^{(2)},$$

where  $m_w^{(2)}$  is the amount of moisture in the gas at the exit from the separator, per unit of gas.

value  $m_w^{(2)}$  using a formula similar to (19), i.e

$$m_w^{(2)} = b_{mc,2} m_{mix} / \rho_{mix}.$$
(21)

After taking into account formulas (19) and (21), we get

$$\Delta m_w = \left(b_{mc,1} - b_{mc,2}\right) m_{mix} / \rho_{mix},$$

or

$$\Delta m_{w} = \Delta b_{mc} m_{mix} / \rho_{mix} , \qquad (22)$$

where  $\Delta b_{mc} = b_{mc,1} - b_{mc,2}$ 

Knowing  $\Delta m_w$ , we can calculate

$$m_{gl} = \frac{\Delta b_{mc} m_{mix} z}{\rho_{mix}},$$
(23)

where  $z = \frac{z_2}{z_1 - z_2}$ 

The obtained values  $m_{mix}, m_w$  and  $m_{gl}$ , make it possible to calculate the mass flow rate of the mixture entering the inlet of the C-2 separator



$$m = m_{mix} + m_w + m_{gl} \, .$$

After taking into account the components  $m_w$  and  $m_{gl}$ , included in the last formula, we get

$$m = m_{mix} \left( 1 + \frac{1}{\rho_{mix}} \left( b_{mc,1} + \Delta b_{mc} z \right) \right).$$

From here

$$m_{mix} = m \left( 1 + \frac{1}{\rho_{mix}} (b_{mc,1} + \Delta b_{mc} z) \right)^{-1}$$

Taking into account that  $b_{mc,1} == a_{m,1}^{(1)} / P_1 + a_{m,2}^{(1)}$  and  $b_{mc,2} = a_{m,1}^{(2)} / P + a_{m,2}^{(2)}$ , we introduce the following notation:  $\theta(P, P_1) = \left(1 + \frac{1}{\rho_{mix}} \left(b_{mc,1} + \Delta b_{mc}z\right)\right)^{-1}$ . Then  $m_{mix} = m\theta(P, P_1).$ (24)

Formula (24) determines the material balance for the gas mixture entering the C-2 separator.

In fig. 2 shows the scheme of material flows of the installation of the second stage of separation, from which it follows that  $m_{g}^{(2)} = m_{g} + m_{w}^{(g)}$ 



Figure. 2. Scheme of material flows of the NTS installation

The gas obtained as a result of separation in quantity  $m_g$  per unit of time should be calculated according to formula (18). By analogy with formula (19), we find

$$m_w^{(g)} = \frac{m_{mix}b_{mc,2}}{\rho_{mix}}$$

Since it  $m_g$  is calculated according to formula (18), then

$$m_g^{(2)} = m_{mix} \left( \left(1 - f_d\right) \sum_{j=1}^{N_c} \frac{\eta_j}{\mu_{w,j}} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s} \mu_{w,i}}{f_d + \left(1 - f_d\right) f_i P^{-1}} + \frac{b_{mc,2}}{\rho_{mix}} \right).$$

Considering formula (22), we have



$$m_g^{(2)} = \theta(P, P_1) \left( \left(1 - f_d\right) \sum_{j=1}^{N_c} \frac{\eta_j}{\mu_{w,j}} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s} \mu_{w,i}}{f_d + (1 - f_d) f_i P^{-1}} + \frac{b_{mc,2}}{\rho_{mix}} \right) m.$$
(25)

Let's enter the following notation:

$$\varepsilon_{g}(P,P_{1}) = \Theta(P,P_{1}) \left( (1-f_{d}) \sum_{j=1}^{N_{c}} \frac{\eta_{j}}{\mu_{w,j}} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s} \mu_{w,i}}{f_{d} + (1-f_{d}) f_{i} P^{-1}} + \frac{b_{mc,2}}{\rho_{mix}} \right).$$

Then (25) will take the following form:

$$m_g^{(2)} = \varepsilon_g \left( P, P_1 \right) m \,. \tag{26}$$

The value  $\varepsilon_g(P, P_1)$  is called the gas separation coefficient [5].

The amount of liquid phase formed as a result of separation is determined by analogy to formula (26). Therefore, (Fig. 2),

$$m_{lq}^{(2)} = m_{con} + m_{dl} + \Delta m_w.$$

The components placed on the right side of the last equation are determined by formulas (17), (22) and (23). Ago

$$m_{lq}^{(2)} = m_{mix} \left( f_d \sum_{j=1}^{N_c} \frac{\eta_j}{\mu_{w,j}} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s} \mu_{w,i}}{f_d + (1 - f_d) f_i P^{-1}} + \frac{\Delta b_{mc} (z+1)}{\rho_{mix}} \right).$$

After taking formula (24) into account, we get

$$m_{lq}^{(2)} = \Theta(P, P_1) \left( f_d \sum_{j=1}^{N_c} \frac{\eta_j}{\mu_{w,j}} \sum_{i=1}^{N_{con}} \frac{\mu_{0i,s} \mu_{w,i}}{f_d + (1 - f_d) f_i P^{-1}} + \frac{\Delta b_{mc} (z+1)}{\rho_{mix}} \right) m \cdot$$

We write the last formula in the following form:

$$n_{lq}^{(2)} = \varepsilon_{lq} \left( P, P_1 \right) m, \qquad (27)$$

where is the coefficient of separation of water from gas [5].

Equations (26) and (27) describe the equilibrium state of the low-temperature separation process, provided that the temperature regime in the separator is unchanged.

The mathematical model of the low-temperature separation process is based on the material balance equation.

If  $M_{sb}$  the mass of the substance that is in the container,  $a m_{ci}$  mass flow fed into the container or leaving it, then

$$\frac{dM_{sb}}{dt} = \sum_{i=1}^{n} m_{ci} \,.$$
(28)

In formula (28), the value  $m_{ci}$  must be taken with a "plus" sign when the flow is directed into the container and with a "minus" sign if the flow leaves the container.

Let's write equation (28) for the liquid in separator C-2 (Fig. 2). We will calculate the value according to the following formula:  $M_{sb}$ 

$$M_{sb} = V_{lp} \rho_{lp},$$

where  $\rho_{lp}$  is the average density of the liquid phase.

The liquid volume  $V_{lp}$  is a function of the liquid level *H* in the container and its appearance depends on the spatial location of the C-2 separator (Fig. 1). Let  $V_0$  the



full volume of the separator. Denote by the  $\nu$  degree of separator filling [9]

$$\mathbf{v}(H) = \frac{V_{lp}(H)}{V_0}.$$

Then

$$M_{sb} = V_0 v(H) \rho_{lp}.$$
<sup>(29)</sup>

The flow of raw materials at the entrance to the C-2 separator (Fig. 1) contains gas, condensate, and moisture, which, as a result of separation, pass into the liquid phase. Formula (3.3) provides a quantitative estimate of the liquid phase.

The consumption of raw materials at the entrance to the C-2 separator (Fig. 1), which is in a gaseous state, will be calculated according to the following formula:

$$m = \alpha \sqrt{K_g \left(P_1 - P\right) P_1}, \qquad (3 0)$$

where  $\alpha$  is the flow rate.

The coefficient  $K_g$  can be calculated using the following expression:

$$K_g = \left(514C\right)^2 \frac{\rho_g}{T},$$

where *C* is a constant value;  $\rho_g$ - gas density at  $t = 20^{\circ}C$ ;  $\rho_g = 0,7 \kappa c / M^3$ ; T-gas temperature <sup>in</sup> K.

Therefore, the arrival of the flow to the separator will be as follows:

$$m_{c1} = \varepsilon_{lq} \left( P, P_1 \right) m,$$

We will calculate the total mass flow rate according to formula (30). Then

$$m_{c1} = \alpha \varepsilon_{lq} (P, P_1) \sqrt{K_g (P_1 - P) P_1}.$$
 (31)

The flow leaving the separator is characterized by mass flow  $m_{c1}$ , which can be calculated using the formula

$$m_{c2} = \alpha_2 (U_2) \sqrt{g \rho_{lp} H + P} , \qquad (32)$$

where  $\alpha_2(U_2)$  is the cost factor, which is a function of the position of the regulatory body.

Taking into account formulas (29) - (32), expression (28) will be as follows:

$$\frac{d\nu(H)}{dt} = \frac{1}{V_0 \rho_{lp}} \Big( \alpha \varepsilon_{lq} (P, P_1) \sqrt{K_g (P_1 - P) P_1} - \alpha_1 (U_1) \sqrt{g \rho_{lp} H + P - P_3} \Big).$$
(33)

Let's find the mass of gas in separator C-2. For a real gas that occupies a volume of  $V_g$ , the ratio is valid

$$PV_g = z_{gc} M_g R_g T , \qquad (34)$$

where  $V_g$  is the gas volume in the separator;  $z_{gc}$  - compressibility coefficient;  $R_g$  - gas table; T - gas temperature.

The calculation of the compressibility coefficients of natural gas  $z_{gc}$  was carried out using the Benedict-Webb-Ravin equation [10] at the gas density under normal conditions [11]  $\rho_{st} = 0.7 \frac{\kappa^2}{M^3}$ . From equation (34) we find  $M_g = \frac{PV_g}{z_{gc}R_gT}$ .

The gas in the C-2 separator occupies only a part of the total volume  $V_0$ , i.e  $V_g = V_0 - V_{lp}$ . Taking into account the value of  $V_{lp} = V_0 v(H)$ , we get  $V_g = V_0 (1 - v(H))$ Taking into account the value of  $V_{lp}$  we find the mass of gas in the C-2 separator

Taking into account the value of  $V_g$ , we find the mass of gas in the C-2 separator

$$M_{g} = \frac{PV_{0}(1 - v(H))}{z_{gc}R_{g}T}.$$
(35)

The gas flow into the C-2 separator can be calculated using formula (26), i.e  $m_{g1} = \varepsilon_g (P, P_1) m$ ,

where m is calculated by formula (30).

So,

$$m_{g1} = \alpha \varepsilon_g \left( P, P_1 \right) \sqrt{K_g \left( P_1 - P \right) P_1} \,. \tag{36}$$

Gas consumption from the C-2 separator to the pipeline will be as follows:

$$m_{g2} = \alpha_1 (U_1) \sqrt{K_g (P - P_2) P},$$
 (37)

where  $\alpha_1(U_1)$  is the cost factor, which is a function of the position of the regulatory body.

Formulas (36) - (37) make it possible to obtain a mathematical model for the gas fraction, i.e

$$\frac{d}{dt}\left(\frac{PV_0(1-\nu(H))}{z_{gc}R_gT}\right) = \alpha\varepsilon_g(P,P_1)\sqrt{K_g(P_1-P)P_1} - \alpha_1(U_1)\sqrt{K_g(P-P_2)P}.$$

We assume that  $z_{gc}$ ,  $R_g$  and T -constant quantities. Then

$$\frac{d\left(P\left(1-\nu(H)\right)\right)}{dt} = \gamma_g \left(\alpha \varepsilon_g \left(P, P_1\right) \sqrt{K_g \left(P_1-P\right) P_1} - \alpha_1 \left(U_1\right) \sqrt{K_g \left(P-P_2\right) P}\right), (38)$$
  
where  $\gamma_g = \frac{z_{gc} R_g T}{V_0}$ 

The degree of filling of the separator depends on the shape of the separator and its position in space.

In equation (33), we calculate 
$$\frac{d\nu(H)}{dt} = \frac{\partial\nu(H)}{\partial H} \cdot \frac{dH}{dt}$$
. Given the formula  

$$\frac{dH}{dt} = \frac{1}{V_0 \rho_{lp} q_p(H)} \Big( \alpha \varepsilon_{lq}(P, P_1) \sqrt{K_g(P_1 - P)P_1} - \alpha_2(U_2) \sqrt{g\rho_{lp}H + P} \Big), \quad (39)$$
where  $\sigma_{lp}(H)$ 

where  $q_p(H) = \frac{\partial v(H)}{\partial H}$ .

Now let's calculate  $\frac{d(P(1-v(H)))}{dt} = (1-v(H))\frac{dP}{dt} - P\frac{dv(H)}{dt}$ . On the other

hand,  $\frac{d(P(1-v(H)))}{dt}$  this is the right-hand side of equation (38). Ago



$$(1-\nu(H))\frac{dP}{dt} = P\frac{d\nu(H)}{dt} + \gamma_g \left(\alpha\varepsilon_g (P,P_1)\sqrt{K_g (P_1-P)P_1} - \alpha_1(U_1)\sqrt{K_g (P-P_2)P}\right).$$
  
Instead  $\frac{d\nu(H)}{dt}$ , let's substitute its value from equation (33), i.e  
$$(1-\nu(H))\frac{dP}{dt} = \frac{P}{V_0\rho_{lp}} \left(\alpha\varepsilon_{lq}(P,P_1)\sqrt{K_g (P_1-P)P_1} - \alpha_2(U_2)\sqrt{g\rho_{lp}H+P}\right) + \gamma_g \left(\alpha\varepsilon_g (P,P_1)\sqrt{K_g (P_1-P)P_1} - \alpha_1(U_1)\sqrt{K_g (P-P_2)P}\right).$$

We write the obtained differential equation in the following form:

$$\frac{dP}{dt} = \frac{1}{1 - \nu(H)} \Big( \gamma_g \Big( \alpha \varepsilon_g (P, P_1) \sqrt{K_g (P_1 - P) P_1} - \alpha_1 (U_1) \sqrt{K_g (P - P_2) P} \Big) + \frac{P}{V_0 \rho_{lp}} \Big( \alpha \varepsilon_{lq} (P, P_1) \sqrt{K_g (P_1 - P) P_1} - \alpha_2 (U_2) \sqrt{g \rho_{lp} H + P} \Big) \Big).$$
(40)

Differential equations (39) and (40) form a mathematical model of the low-temperature separation process.

Mathematical models (39) and (40) are nonlinear with respect to input and output values. We assume that the means of automation provide small deviations of the regulated values from the given (established) values. In such a case, the nonlinear functions included in models (39) and (40) can be expanded into a Taylor series, keeping only the linear terms of the expansion.

We will mark fixed values of input and output values with superscripts "0". Let  $H = H^{(0)} + \Delta H$ ,  $P = P^{(0)} + \Delta P$ ,  $U_1 = U_1^{(0)} + \Delta U_1$ ,  $U_2 = U_2^{(0)} + \Delta U_2$ ,  $P_1 = P_1^{(0)} + \Delta P_1$ .  $P_2 = P_2^{(0)} + \Delta P_2$ 

system of differential equations (39) and (40) in vector form

$$\frac{d\overline{X}}{dt} = \overline{\varphi} \Big( \overline{X}, \overline{U}, \overline{\zeta} \Big), \tag{41}$$

where 
$$\overline{X} = \begin{bmatrix} H \\ P \end{bmatrix}$$
 is the vector of initial values;  $\overline{U} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$  - vector of control actions;  
 $\overline{\zeta} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$  - disturbance vector;  $\overline{\varphi}(\overline{X}, \overline{U}, \overline{\zeta}) = \begin{bmatrix} \varphi_1(\overline{X}, \overline{U}, \overline{\zeta}) \\ \varphi_2(\overline{X}, \overline{U}, \overline{\zeta}) \end{bmatrix}$  - the vector function of the

right parts of the system of differential equations (39) and (40).

If the nonlinear function  $\overline{\varphi}(\overline{X}, \overline{U}, \overline{\zeta})$  is expanded into a Taylor series, limiting itself only to the linear terms of the expansion, then

$$\frac{d\overline{x}}{dt} = A\overline{x} + B\overline{u} + \Gamma\overline{\zeta}, \qquad (42)$$

$$\overline{y} = C\overline{x} , \qquad (43)$$

where 
$$A = J_{\varphi x} \left( \overline{X}^{(0)}, \overline{U}^{(0)}, \overline{\zeta}^{(0)} \right)_{-} \quad B = J_{\varphi u} \left( \overline{X}^{(0)}, \overline{U}^{(0)}, \overline{\zeta}^{(0)} \right); \quad \Gamma = J_{\varphi \zeta} \left( \overline{X}^{(0)}, \overline{U}^{(0)}, \overline{\zeta}^{(0)} \right)^{-}$$



matrices of partial derivatives (Jacobi matrices);  $\overline{x} = \Delta \overline{X}$ ;  $x_1 = \Delta H$ ;  $x_2 = \Delta P$ ;  $\overline{u} = \Delta \overline{U}$ ;

$$u_1 = \Delta U_1; \ u_2 = \Delta U_2; \ \overline{\zeta} = \Delta \overline{\zeta}; \ \zeta_1 = \Delta P_1; \ \zeta_2 = \Delta P_2; \ \overline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
- vector of state variables;

 $\overline{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \text{-vector of control actions; } \overline{\zeta} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} \text{-disturbance vector; } \overline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \text{-vector of }$ 

initial values; C - unit matrix.

Equations (42) and (43) are a linearized mathematical model of the lowtemperature separation process, in which the elements of the matrices A, B and  $\Gamma$  are calculated at fixed values  $\overline{X} = \overline{X}^{(0)}, \overline{U} = \overline{U}^{(0)}$  and  $\overline{\zeta} = \overline{\zeta}^{(0)}$ \_\_\_\_\_\_

Since the differential equations are written in increments of relatively constant values, the initial conditions are zero.

The calculation of the parameters of the linearized mathematical model was carried out with the following input data: the total flow of gas entering the separation - m = 2.3576 kg/s; the amount of condensate released  $m_{con} = 1.1574 \text{ kg/s}$ ; gas pressure before the C-2 separator (Fig. 1)  $P_1 = 13.6 MPa$ ; pressure in separator C-2 P = 5.3 MPa; pressure at the entrance to the third stage of separation  $P_2 = 1,2 MPa$ ; gas temperature at the inlet to the separator C-2  $T_{in} = 17 \text{ }^{\circ}C$ ; temperature in separator C-2  $T = -21^{\circ}C$ ; the dew point of the gas in terms of humidity at a pressure of 4MPa no more than  $4^{\circ}C$ ; the diameter of the low-temperature separator  $d_s = 1.0m$ ; condensate level in the separator  $H^{(0)} = 0.6m$ .

following relationship exists between the diameter of the separator and its length  $d_s$  [12]:  $\frac{L_s}{d_s}$  = 3 Since the pressure in separator 4 is 5.3*MPa*, the gas behavior in the separator will be determined by formula (34). Using the software created in the MatLab environment, the gas compressibility coefficient was calculated: Z= 0.8150.

The moisture content of the gas will be determined by the Byukachek formula. The coefficients  $a_{m1}$  and  $a_{m2}$  according to [13] are as follows:

$$a_{m1} = \frac{0,457}{1,42} \cdot \exp(0,0735t - 0,27 \cdot 10^{-3}t^{2}),$$
  
$$a_{m2} = 0,0418 \cdot \exp(0,054t - 0,2 \cdot 10^{-3}t^{2}),$$

where is t-the temperature  $^{\circ}C$ 

will  $a_{m2}$  determine by known coefficients  $a_{m1}$ 

$$\Theta(P,P_1) = \left(1 + \frac{1}{\rho_{mix}} \left(b_{mc,1} + \Delta b_{mc}z\right)\right)^{-1},$$
  
where  $b_{mc,1} = \frac{a_{m1}^{(1)}}{P_1} + a_{m2}^{(1)} \Delta b_{mc} = \frac{a_{m1}^{(1)}}{P_1} - \frac{a_{m1}^{(2)}}{P_1} + a_{m2}^{(1)} - a_{m2}^{(2)}.$ 

We accept [5]: z = 0, 2;

The chemical composition of the gas entering the low-temperature separation is borrowed from the work [14], using these data, the molar mass of the gas was determined by the formula:  $q_M = \frac{1}{100} \sum_{i=1}^{n_u} \mu_{q,i} v_{q,i}$ , where  $\mu_{q,i}$  is the molar mass of the i-th component of the gas, kg/kmol;  $v_{q,i}$ - volume fractions of gas components, %. After calculations, we got:  $q_M = 20.0221 \frac{kg}{kmol}$ . The density of natural gas under normal physical conditions will be as follows [11] :  $\rho_0 = \frac{q_M}{22,4}$  Taking into account the obtained value  $q_M$ , we find that  $\rho_0 = 0.8325 \frac{kg}{m^3}$  The gas density for separation conditions can be calculated using the formula [15] :  $\rho_g = \rho_0 \frac{273P}{0,1013 \cdot (273+T)}$ , where P is the gas pressure in the separator, MPa; T - the temperature of the gas in the separator, °C. Taking into account the values of P and T, we get:  $\rho_g = 47.1871 \frac{kg}{m^3}$ 

The consumption of gas leaving the separator is defined as the difference between the total consumption m and the condensate consumption  $m_{con}$ , i.e.  $m_g = m - m_{con}$  Taking into account the values m and  $m_{con}$  obtained:  $m_g = 1.2202 \frac{kg}{s}$ . We determine the number of moles (per second) that went to separation  $M = \frac{m}{a_{vo}}$ 

After substituting numerical values into *m* the  $q_M$  last formula, we obtained:  $M = 0.1177 \frac{kmol}{s}$ Number of moles of gas formed as a result of separation:  $M_g = \frac{m_g}{q_M}$ . Since the numerical values of  $m_g$  and are known  $q_M$ , then:  $M_g = 0.0609 \frac{kmol}{s}$ . From formula (3.9) we find:  $f_d = 1 - \frac{M_g}{M}$ .

According to the known values  $M_g$ , M the following result was obtained:  $f_d = 0.4824$ .

Formula (39) can also be written for the mass flow rate of the *m* gas mixture:  $M = m \sum_{i=1}^{N_c} \frac{\eta_i}{\mu_{w,i}}$ ,

where M - is the number of moles per second. From the last equation we determine  $\sum_{i=1}^{N_c} \frac{\eta_i}{\mu_{w,i}} = \frac{M}{m}$ After substituting the corresponding values in the last formula, we get  $q_f = \sum_{i=1}^{N_c} \frac{\eta_i}{\mu_{w,i}} = 0.0499 \frac{kmol}{kg}.$ 

Gas condensate is a liquid that is separated from natural gas in the process of low-temperature separation, and is a mixture of hydrocarbons that contain more than



4 carbon atoms.

It was assumed that the mixture of gases and condensate obey the laws of ideal solutions. Ago

$$\frac{f_i}{P} = 1.$$

Taking into account the assumption made, the formula for determining  $\varepsilon_{lq}$ , will take the following form:

$$\varepsilon_{lq}(P,P_1) = \Theta(P,P_1) \left( f_d q_f \sum_{i=1}^{N_{con}} \mu_{0i,s} \mu_{w,i} + \frac{\Delta b_{mc}(z+1)}{\rho_{mix}} \right).$$

Let's introduce the following notation:

$$S_{\mu} = \sum_{i=1}^{N_{con}} \mu_{0i,s} \mu_{w,i} \,.$$

Taking into account the accepted notation, we will have

$$\varepsilon_{lq}(P,P_1) = \Theta(P,P_1) \left( f_d q_f S_{\mu} + \frac{10^{-3} \cdot \Delta b_{mc}(z+1)}{\rho_{mix}} \right).$$
(44)

After the calculations, we got the following value:

$$S_{\mu} = 78,862 \, \frac{kg}{kmol}.$$

Since the moisture content, which is calculated according to Bukachek's formula, has a dimension of  $g / m^3$ , then in order to reconcile the dimension of the moisture content  $b_{mc}$  with the dimension of the density  $\rho_{mix}$  in formula (44) the multiplier is entered 10<sup>-3</sup>. Then the moisture content will have the dimension  $\frac{kg}{m^3}$ .

We will obtain a similar formula for determining the gas separation coefficient

$$\varepsilon_g(P,P_1) = \Theta(P,P_1) \left( (1-f_d) q_f S_\mu + \frac{10^{-3} \cdot b_{mc,2}}{\rho_{mix}} \right).$$
(45)

Now let's move on to the determination of the numerical values of the values of the models of the dynamics of the separation process.

Let's write equation (30) for the steady state (superscripts "0" denote values of values in the steady state). We have  $\alpha \sqrt{K_g (P_1^{(0)} - P^{(0)}) P_1^{(0)}} = m$ :

From the last equation we find:  $\alpha_K = \alpha K_g^{1/2} = \frac{m}{\sqrt{\left(P_1^{(0)} - P^{(0)}\right)P_1^{(0)}}}$ .

Taking into account the numerical values of the corresponding quantities, we will have:  $\alpha_{K} = 0.1906 \frac{kg}{s \cdot MPa}$ .

Static characteristics of regulatory bodies  $\alpha_i(U_i)$ , i=1,2 approximate by linear dependences  $\alpha_i(U_i) \approx \alpha_{U,i}U_i$ , i=1,2. Formula (32) and (37) will be written for the



steady state

$$m_{g2} = \alpha_1 \left( U_1^{(0)} \right) K_g^{1/2} \sqrt{\left( P^{(0)} - P_2^{(0)} \right) P^{(0)}}, \qquad (46)$$

$$m_{c2} = \alpha_2 \left( U_2^{(0)} \right) \sqrt{g \rho_{lp} H^{(0)} + P^{(0)}} , \qquad (47)$$

From equation (46) we find

$$\alpha_{G} = \alpha_{1} \left( U_{1}^{(0)} \right) K_{g}^{1/2} = \frac{m_{g2}}{\sqrt{\left( P^{(0)} - P_{2}^{(0)} \right) P^{(0)}}}.$$

We assume that  $U_1^{(0)} = 0,06MPa$ .

Taking into account the linear dependence of the static characteristic of the regulatory body, we obtain the following equation:  $\alpha_G = \alpha_{U,I} U_1^{(0)}$ . From here we find

$$\alpha_{U,1} = \frac{\alpha_G}{U_1^{(0)}} \cdot$$

Taking into account the corresponding numerical values, we will have:  $\alpha_{U,1} = 4.3626 \frac{kg}{s \cdot (MPa)^2}.$ 

Let's solve equation (3.24) with respect to  $\alpha_{con} = \alpha_2 \left( U_2^{(0)} \right) = \frac{m_{c2}}{\sqrt{g\rho_{lp}H^{(0)} + P^{(0)}}}$ .

Since 
$$\alpha_{con} = \alpha_{U,2} U_2^{(0)}$$
, then  $\alpha_{U,2} = \frac{\alpha_{con}}{U_2^{(0)}}$ . We accept:  $U_1^{(0)} = U_2^{(0)} = 0.06 MPa$ 

The density of condensate is as follows [14] :  $\rho_{lp} = 743.6 \frac{kg}{m^3}$ .

After taking into account the corresponding numerical values, the following result was obtained:  $\alpha_{U,2} = 8.2005 \frac{kg}{s \cdot (MPa)^2}$ .

Now let's find the numerical value  $\gamma_g$ , which is calculated according to the following formula:  $\gamma_g = \frac{z_{gc}R_gT}{V_0}$ . Since the container has a cylindrical shape, then  $V_0 = \pi r_s^2 L_s$ .

So,  $\gamma_g = 33,0448 \ 1/(m \cdot s^2)$ .

After the assumptions made and the notations entered, the mathematical model of the dynamics of the low-temperature separation process will be as follows:

$$\frac{dH}{dt} = \frac{1}{V_0 \rho_{lp} q_p(H)} \Big( \alpha_K \varepsilon_{lq}(P, P_1) \sqrt{(P_1 - P)P_1} - \alpha_{U,2} U_2 \sqrt{g \rho_{lp} H + P} \Big);$$

$$\frac{dP}{dt} = \frac{1}{1 - \nu(H)} \Big( \gamma_g \Big( \alpha_K \varepsilon_g(P, P_1) \sqrt{(P_1 - P)P_1} - \alpha_{U,1} U_1 \sqrt{(P - P_2)P} \Big) +$$
(48)



$$+\frac{P}{V_{0}\rho_{lp}}\left(\alpha_{K}\varepsilon_{lq}(P,P_{1})\sqrt{(P_{1}-P)P_{1}}-\alpha_{U,2}U_{2}\sqrt{g\rho_{lp}H+P}\right)\right),$$
(49)  
where  $\varepsilon_{lq}(P,P_{1}) = \Theta(P,P_{1})\left(f_{d}q_{f}S_{\mu}+\frac{10^{-3}\cdot\Delta b_{mc}(z+1)}{\rho_{mix}}\right) - \varepsilon_{g}(P,P_{1}) = \Theta(P,P_{1})\left((1-f_{d})q_{f}S_{\mu}+\frac{10^{-3}\cdot b_{mc,2}}{\rho_{mix}}\right).$ 

Mathematical models (48) and (49) were linearized relative to the established values of  $H^{(0)}$ ,  $P_1^{(0)}$ ,  $P_2^{(0)}$ ,  $U_1^{(0)}$  and  $U_2^{(0)}$ . For this, we used a program created in the MatLab environment. After calculations, we got the following result:

PARAMETERS OF THE LINEARIZED MATHEMATICAL MODEL Elements of matrix A a11=-1.8075e-02 a12=5.5433e-05 a21=1.1670e+03 a22=-4.9400e+01

Elements of matrix B b11=0.0000e+00 b12=2.4836e-01 b21=1.9944e+03 b22=4.3962e+00

Elements of the Gama matrix gm11=6.4585e-05 gm12=0.0000e+00 gm21=3.8864e+01 gm22=1.4593e+011

Let's find the matrix transfer functions of the low-temperature separation process. To do this, let's transform the linearized mathematical model (42) and (43) according to Laplace with zero initial conditions. So,

$$\overline{Y}(p) = C(Ip - A)^{-1}(B\overline{U}(p) + \Gamma\overline{\varsigma}(p)).$$
(50)

Analysis of expression (50) shows that the dynamic properties of the separator, as an object of automatic control, is characterized by two matrix transfer functions: relative to the vector of control actions

$$W_{yu}(p) = C(Ip - A)^{-1}B$$
(51)

and relative to the perturbation vector

$$W_{y\zeta}(p) = C(Ip - A)^{-1}\Gamma$$

If the matrix transfer function is known  $W_{yu}(p)$ , then

$$Y(p) = W_{yu}(p)U(p).$$

Since C is a unit matrix, then

$$W_{yu}(p) = (Ip - A)^{-1} B.$$
(52)

From the matrix equation (52), we find



$$w_{11}(p) = \frac{b_{11}p + a_{12}b_{21} - a_{22}b_{11}}{\Delta(p)}, \ w_{12}(p) = \frac{b_{12}p + a_{12}b_{22} - a_{22}b_{12}}{\Delta(p)},$$
$$w_{21}(p) = \frac{b_{21}p + a_{21}b_{11} - a_{11}b_{21}}{\Delta(p)}, \ w_{22}(p) = \frac{b_{22}p + a_{21}b_{12} - a_{11}b_{22}}{\Delta(p)}.$$

Based on the found transfer functions  $w_{ij}(p)$ , we will i, j = 1, 2 determine the value of the output values of the separator relative to the control actions

$$Y_{1}(p) = w_{11}(p)U_{1}(p) + w_{12}(p)U_{2}(p),$$
(53)

$$Y_{2}(p) = w_{21}(p)U_{1}(p) + w_{22}(p)U_{2}(p).$$
(54)

Taking into account the numerical data, the following transfer functions of the low-temperature separator were obtained:

$$w_{11}(p) = \frac{0.1106}{p^2 + 49.42p + 0.8282}; \ w_{12}(p) = \frac{0.2484p + 12.27}{p^2 + 49.42p + 0.8282};$$

$$w_{21}(p) = \frac{1994p + 36.05}{p^2 + 49.42p + 0.8282}; \ w_{22}(p) = \frac{4.396p + 289.9}{p^2 + 49.42p + 0.8282}$$

Based on equations (53) and (54), a structural diagram of the separator as an object of automatic control was created (Fig. 3), from which it can be seen that there are cross-connections between inputs and outputs, which negatively affect the quality of the control process.



## Figure. 3. Structural diagram of a low-temperature separator

Thus, the separator, as an object of automatic control, belongs to the class of multidimensional objects with internal cross connections, which complicates the task of automatic control of such an object. Therefore, the task of further research is the synthesis of an autonomous control system, which will make it possible to compensate for cross-connections and thereby significantly improve the quality of the control process.

# Conclusions

1. On the basis of the law of conservation of the amount of matter, a mathematical model of the low-temperature separation process in terms of "input-output" was created, which is suitable for control purposes.

2. The linearization of the mathematical model of the low-temperature separation process was carried out under the condition that the deviations of the output values from the established values are insignificant, which made it possible to obtain the transfer functions of the controlled object and, based on their analysis, the presence of cross-connections between the input and output values was revealed , which is the basis for the synthesis of an automatic control system for the low-temperature separation process.

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