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INFLUENCE OF STIFFNESS ELLIPSES ORIENTATION AND DISSIPATION CHARACTERISTICS ON THE LATHE DYNAMIC SYSTEM STABILITY DURING CUTTING PROCESS

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Abstract. The article presents the theoretical studies results of direction influence of reduced stiffness and damping parameters in the machine tools dynamic system called as "cutter tool-workpiece" during turning. The influence determination and compilation of reduced stiffness and damping matrices in the "cutter tool" subsystem. When designing tool holder with oriented center of rigidity for the lathe machine with the given elastic parameters of the carriage there is a question of rational choice of equipment elastic and damping parameters, with usage of which would increase the vibration stability. The issue of determining the effectiveness of the use of additional equipment with the aim of improving vibration resistance for turning potentially unstable dynamic system carriage of the machine tools with cutting requires the creation of a common mathematical model of the machine dynamic system its modeling and definition of vibration treatment stock in various elastic parameters of the system equipment.

In accordance with the received analytical dependencies formulated conditions asymptotic sustainability dynamic systems. Defined conditions for effective raise level vibration resistance through differential systems equations scalarization in dynamics. Design implementation given principles raise vibration resistance embodied in the form received patents of special designed lathe tool holders.

To ensure the asymptotic stability of the dynamic system of metal-cutting machine tools during cutting, an important area for improvement is the machine tools carriage and properties of the cutting tool. It is important to make such design changes in the design of the machine tools that allow you to influence the dissipative and elastic parameters orientation of the dynamic subsystem of the cutter-carriage.

The theoretical explanations presented in the article substantiate the directions for vibration resistance increasing of turning processing due to the negative influence of the "coordinate links" reduction between the cutter tools vibrational movement and the cutting process, due to the use of an additional tool holder with an oriented position of the center of rigidity or a cutter with increased damping characteristics.

Key words: tool holder, vibrostability, lathe work, asymptotic sustainability, the dynamics of machine tools, center of rigidity.

Introduction.

The problem of vibration resistance increasing of the machine tools is of great practical importance. The lathe's productivity at a given machining accuracy is largely determined by the quality of the lathe's carriage and its vibration resistance. According to the "Coordinate Links Theory", the physical essence of the machine tools dynamic systems constancy loss with the self-oscillations subsequent occurrence during turning processing is explained by the fact that when the cutter tool moves in the vector of cutting force actions direction, with machine tool holders certain elastic parameters, the metal chip's thickness becomes larger, than when moving in the opposite direction. The cutting force increases as the slice thickness



increases and decreases as it decreases. Therefore, this ambiguity of the change in the cutting force due to the cutter tools movement relative to the workpiece leads to establishment and increasing of self-oscillations. The phenomenon of dynamic stability loss is especially evident in machine tools with a potentially unstable lathe carriage system, namely, when the elastic characteristics of base machine tools carriage system has an irrational orientation of the main stiffness axes, for example, during two-sided machining, or when the carriage system does not have clearly defined stiffness axes.

Main text

PROBLEM STATEMENT. In the studies [1, 3, 4, 5] of V.A. Kudinov and J. Tlustý found that the machine tools vibration resistance depends not only on its elastic systems stiffness, but also on the orientation of the stiffness main axes of relative to the direction of cutting force actions vector. Also, at the present time, several constructive solutions of tool holders with oriented center of rigidity are known, when using which it was possible to significantly increase the machine tools vibration resistance. However, today it is expedient to carry out theoretical studies of the effect of cutter tool holders elastic parameters, which is to some extent related to the center of rigidity position and the parameters of reduced stiffness values along the main axes on the dynamic systems "cutter tool-workpiece" vibration resistance of during cutting processing.

The lathe carriage elastic system, as a complex dynamic system with many degrees of freedom, has many its own oscillations forms, each of which can be distinguished by ellipses of movements, the main axes directions of which do not coincide with the general axes of the machine tools coordinates. Intense self-oscillations are carried out at a frequency that corresponds to the natural oscillations frequency of the dominant elastic system, namely to the oscillation frequency of the link that has the largest displacement ellipses dimensions. Considering this fact, it is possible to mathematically represent the carriage dynamic system of with the installed cutter tool holder and cutter as a one-mass system with two degrees of freedom. Such a simplification of a multi-link system is permissible, since oscillations at other frequencies during cutting are not dominant and have minor "coordinate links" effects on the machining process. A simplified one-mass mathematical model of a potentially unstable carriage system allows for theoretical studies of the stability loss, since Nyquist diagram of this system, which consists of the frequency (positive and negative) each of the two normal oscillations forms characteristics, crosses the negative real axis.

MATERIAL AND RESULTS. We will use the ideas of V.A. Kudinov about the stiffness ellipses orientation of the carriage subsystem [4]. According to these ideas, in the carriage assemblies group there always exist two such orthogonal axes, in the direction of which the movements correspond only to the axes direction of their orientation.

Then the equations describing the dynamics of the system can be represented according to V.A. Kudinov (Figure. 1) as follows:



$$\begin{cases} m_1 \ddot{\zeta}_1 + h_1 \dot{\zeta}_1 + c_1 \zeta_1 = P \cos \beta \\ m_2 \ddot{\zeta}_2 + h_2 \dot{\zeta}_2 + c_2 \zeta_2 = P \cos \beta' \end{cases} \quad (1)$$

where m_1 and m_2 - reduced to the corresponding coordinates of the dynamic system mass; h_1 and h_2 - reduced damping coefficients.

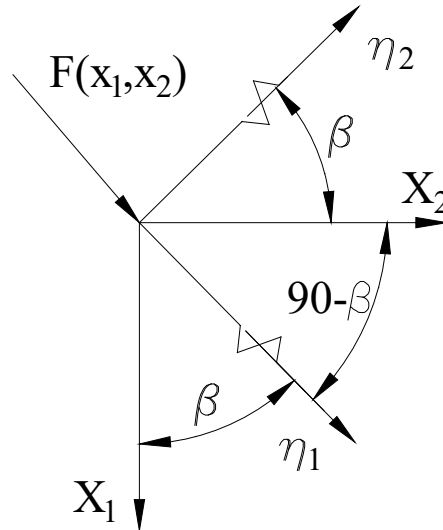


Figure 1 – The influence scheme of the cutter tools elastic deformations in the way of coordinates X_1 and X_2

Then the equation of the dynamic system with respect to the equilibrium point can be represented as

$$\begin{cases} m_1 \frac{d^2 \eta_1}{dt^2} + h_{01} \frac{d\eta_1}{dt} + C_{01} \eta_1 = F_{\eta_1} \\ m_2 \frac{d^2 \eta_2}{dt^2} + h_{02} \frac{d\eta_2}{dt} + C_{02} \eta_2 = F_{\eta_2} \end{cases}, \quad (2)$$

In equations (2), the forces F_{η_1} and F_{η_2} are considered given, and they are not represented by the system coordinates in the directions x_1 and x_2 .

Let us take into account that the dynamic characteristic of the cutting process is formed in the coordinates x_1 and x_2 , therefore equation (2) must be converted to the coordinates x_1 and x_2 , which describe the cutter tool elastic deformation forces and are tied to the machine tools base. It is in the coordinates that x_1 and x_2 dimensional error of turning processing, shape accuracy, roughness, etc. are formed.

Solving equations (2) and expressing η_1 , η_2 and F_{η_1} , F_{η_2} , we get:

$$\begin{aligned} \eta_1 &= x_1 \cos \beta + x_2 \sin \beta; F_{\eta_1} = F_1 \cos \beta + F_2 \sin \beta \\ \eta_2 &= -x_1 \sin \beta + x_2 \cos \beta; F_{\eta_2} = -F_1 \sin \beta + F_2 \cos \beta \\ \begin{cases} C_{01}(x_1 \cos \beta + x_2 \sin \beta) = C_{01} \eta_1 = F_1 \cos \beta + F_2 \sin \beta \\ C_{02}(-x_1 \sin \beta + x_2 \cos \beta) = C_{02} \eta_2 = -F_1 \sin \beta + F_2 \cos \beta \end{cases} \end{aligned}$$



Subtract the second term from the first term. Then we multiply the first term of the equation written in brackets by $\cos\beta$, and the second - by $\sin\beta$:

$$\begin{cases} C_{01}(x_1 \cos \beta + x_2 \sin \beta) \cos \beta = F_1 \cos^2 \beta + F_2 \sin \beta \cos \beta \\ C_{02}(-x_1 \sin \beta + x_2 \cos \beta) = -F_1 \sin^2 \beta + F_2 \cos \beta \sin \beta \end{cases}$$

Let's sum up the equations:

$$\begin{aligned} F_1 &= C_{01}(x_1 \cos^2 \beta + x_2 \sin \beta \cos \beta) + C_{02}x_1 \sin^2 \beta - C_{02}x_2 \sin \beta \cos \beta = \\ &= (C_{01} \cos^2 \beta + C_{02} \sin^2 \beta)x_1 + (C_{01} \sin \beta \cos \beta + C_{02} \sin \beta \cos \beta)x_2; \end{aligned}$$

$$F_1 = C_{01}(x_1 \cos^2 \beta) + C_{02}(x_1 \sin^2 \beta) + (C_{01} - C_{02} \sin \beta \cos \beta)x_2;$$

$$C_{01}(x_1 \sin \beta \cos \beta + x_2 \sin^2 \beta) = F_1 \sin \beta \cos \beta + F_1 \sin^2 \beta;$$

$$C_{02}(-x_1 \sin \beta \cos \beta + x_2 \cos^2 \beta) = -F_1 \sin \beta \cos \beta + F_2 \cos^2 \beta;$$

$$((C_{01} - C_{02}) \sin \beta \cos \beta)x_1 + (C_{01} \sin^2 \beta + C_{02} \cos^2 \beta)x_2 = F_2.$$

After the transformation of the equations, we obtain the arguments for the stiffness matrix C_{11} ; C_{12} ; C_{21} ; C_{22} :

$$C = \begin{bmatrix} C_{01} \cos^2 \beta + C_{02} \sin^2 \beta & (C_{01} - C_{02}) \sin \beta \cos \beta \\ (C_{01} - C_{02}) \sin \beta \cos \beta & C_{01} \sin^2 \beta + C_{02} \cos^2 \beta \end{bmatrix} = \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix}. \quad (3)$$

Thus, in the coordinate system \mathbf{x}_1 and \mathbf{x}_2 we have the following stiffness matrix:

$$\begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix}$$

Note that the resulting stiffness matrix is no longer a diagonal matrix, so the forces acting in the direction x_1 cause deformative mixing, as in the direction of the axes x_1 and x_2 .

At the same time, we note that after rotation of the coordinates, the structural properties of the matrix remain unchanged. It is easy to show that expressions (3) are positive definite for $m > 0$, $h_i > 0$, $C_i > 0$ ($i=1,2$). In addition, matrix (3) is initially symmetric and remains symmetric after its rotation.

The dissipation (damping coefficient) matrix can be represented in a similar way [h], in particular, if the ellipses orientations damping coefficients h_{01} and h_{02} coincide with the orientations η_1 and η_2 , then the dissipation matrix is expressed as follows:

$$h = \begin{bmatrix} h_{01} \cos^2 \beta + h_{02} \sin^2 \beta & (h_{01} - h_{02}) \sin \beta \cos \beta \\ (h_{01} - h_{02}) \sin \beta \cos \beta & h_{01} \sin^2 \beta + h_{02} \cos^2 \beta \end{bmatrix}. \quad (4)$$



Obviously, the matrix also retains its structural properties.

Let us analyze matrices (3) and (4). Off-diagonal elements vanish if one of two conditions is met:

- angle $\beta=0$, which actually means that the stiffness ellipse has diagonal elements that coincide with the axes x_1 and x_2 ;
- $C_{01}=C_{02}$, i.e. the directional stiffnesses η_1 and η_2 are equal, and matrix (3) again becomes diagonal.

The same can be said about the dissipation matrix (4). Note that the "diagonalization" of matrices (3) and (4) leads to scalarization of the original system of differential equations in the considered input equations with respect to the equilibrium point.

The scalarization of equations makes it possible to eliminate the mutual connection between elastic deformation displacements in the direction x_1 and x_2 , thereby, significantly increase the dynamic stability of the system.

Let us show this for the case when the kinematic characteristic of the cutting process can be ignored.

If the scalarization condition is satisfied (the diagonal of the matrix is symmetric and equals zero), then the equation in variations:

$$m_1 \frac{d^2 x}{dt^2} + h \frac{dx}{dt} + Cx = \varphi(x_2, \frac{dx_2}{dt}). \tag{5}$$

As a result of elastic deformations, the influence of dissipative forces in the tool holder, as well as the inertial forces of the tool tip movement, we obtain three matrices of the cutting process in variations relative to the equilibrium point:

$$[m] = \begin{bmatrix} m_{11} & 0 \\ 0 & m_{22} \end{bmatrix}; [h] = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix}; [C] = \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix};$$

$$\varphi = \left\{ \varphi_1(x_2, \frac{dx_2}{dt}), \varphi_2(x_2, \frac{dx_2}{dt}) \right\}^T.$$

So the function vector dynamic response process cutting obtained in variations with respect to the equilibrium point.

It has the following properties:

- 1) if x_2 tends to zero, then j_1 and j_2 also tend to zero;
- 2) following the theory of asymptotic stability to determine the stability of system (5) in variations with respect to the equilibrium point, we analyze the linearized equation (5), i.e. instead of $j_1(x)$, equation (5) will be solved by the first approximation method.

If the scalarization condition is satisfied, then the equation in variations (5) is transformed into the following equation in vector form:

$$m_1 \frac{d^2 x_1}{dt^2} + h_{11} \frac{dx_1}{dt} + C_{11} x_1 = \varphi_1(x_2, \frac{dx_2}{dt}),$$

$$m_2 \frac{d^2 x_2}{dt^2} + h_{22} \frac{dx_2}{dt} + C_{22} x_2 = \varphi_2(x_2, \frac{dx_2}{dt}). \tag{6}$$



In this case, there is no coordinate in the second equation (6) x_1 , then the stability condition in the variations of the relativity of the equilibrium point will be determined by the system

$$m_2 \frac{d^2 x_2}{dt^2} + \left(h_{22} - \frac{d\varphi_2}{\partial \left(\frac{dx_2}{dt} \right)} \right) \frac{dx_2}{dt} + \left(C_{22} - \frac{d\varphi_2}{dx_2} \right) x_2 = 0. \tag{7}$$

Thus, the asymptotic stability condition is determined by the following expression:

$$\begin{cases} h_{22} - \frac{d\varphi_2}{\partial \left(\frac{dx_2}{dt} \right)} > 0; \\ C_{22} - \frac{d\varphi_2}{dx_2} > 0. \end{cases} \tag{8}$$

Rigidity in the process of cutting in the direction C_{22} should be the greatest.

Under the condition of stability of the second equation in system (8) in the stationary state, $x_2 - \frac{dx_2}{dt} = 0$ the properties of the first equation are determined based on the equation:

$$m_1 \frac{d^2 x_1}{dt^2} + h_{11} \frac{dx_1}{dt} + C_{11} x_1 = 0. \tag{9}$$

The equation on the right is zero, because φ_1 from zero is identically equal to zero. System (9) is by definition asymptotically stable.

The analysis showed that in order to ensure the asymptotic stability of the cutting process, an important direction in improving the support group of the machine tool, including the properties of the cutting tool, are such design changes that allow you to influence the dissipative and elastic systems orientation of the cutting tool subsystem.

Obviously, under the condition, if we additionally take into account the influence of the cutting process kinetic characteristics, i.e. dependence of forces $\mathbf{P} \frac{dx}{dt}$ on time, then the first equation will have the following form:

$$m_1 \frac{d^2 x_1}{dt^2} + \left(h_{11} - h_{11}^{(p)} \right) \frac{dx_1}{dt} + C_{11} x_1 = 0. \tag{9}$$

Then an additional requirement for asymptotic stability will be :

$$h_{11} > h_{11}^{(p)}.$$

Under all conditions, asymptotic stability is ensured by increasing parameters C_{22} , h_{22} and h_{11} .

Summary and conclusions.

To ensure the asymptotic stability of the system as a whole, the following design changes in the cutting tool are required. It is necessary to design the tool holder in such a way as to vary its elastic and dissipative (damping) properties in space, i.e. try to create a finite-dimensional dynamic structure of the tool holder (Figure. 2).

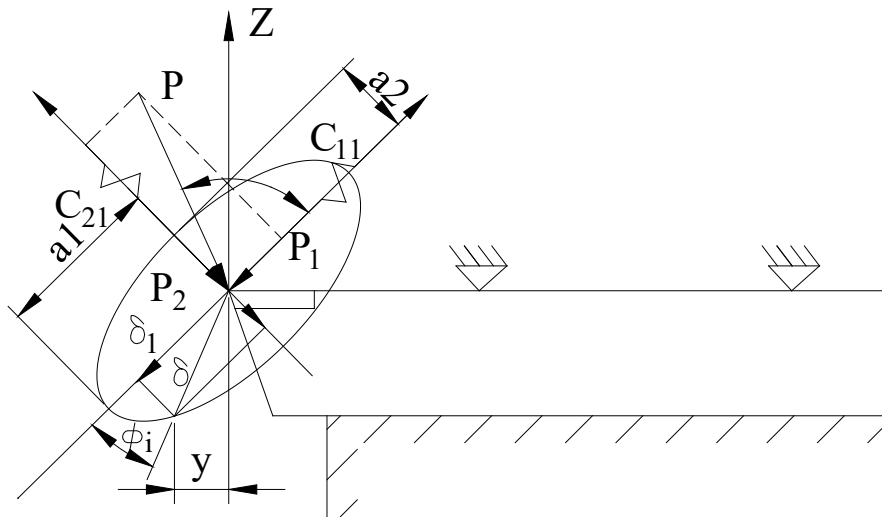
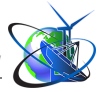


Figure 2 – The movements flat chart of the vertices of a special cutter tool holder design with oriented rigidity (workpiece not shown)

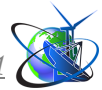
Therefore, resistance of the technological vibration system depends not only on the values of the system's main parameters of (masses, stiffness coefficients and damping elements), but also on the main stiffness axes orientation relative to the the cutting force vector direction.

The existing approaches to the analysis of the dynamic stability of the cutting process are based on the concept of the dependence of the cutting force on the elastic displacements of the tool relative to the workpiece in the direction to the normal of the cutting surface. Such ideas are reflected in the works [1, 4, 6], and others. At the same time, the transformation of the entire dynamic structure of the machine tool depending on the space coordinates and its state is not disclosed.

Conditions for the asymptotic stability of the system are determined. It is shown that one of the effective conditions for improving stability is the scalarization of equations systems in dynamics. To implement this principle, patents for the design of turning tool holders have been proposed and received. Based on an analytical study of the conditions for the asymptotic stability of the tool subsystem and taking into account the directions of its maximum rigidity and dissipation, designs of tool holders with directional stiffness, as well as tool holders with an oriented position of the center of rigidity, have been developed that make it possible to reduce the influence of coordinate interaction and, on this basis, increase the vibration resistance of the turning process [6].

In addition, experimental studies on comparative turning with tool holders have shown that the efficiency of neutralizing the coordinate relationship increases with a decrease in the damping properties of the tool holder with directional rigidity. When using a tool holder with directional rigidity [7], vibration resistance increases by 2.5 - 3 times, and when using tool holders with damping elements - 1.2 - 1.5 times in relation to the cutter tools installed in standard steel tool holders.

The solution to the problem of increasing the vibration resistance of machining is the use of a cutter holder with an oriented position of the center of rigidity, which corrects the ellipse of the movement of the carriage dominant dynamic system at its



main oscillation frequency and thus allows reducing the “coordinate links” influence. Since the carriage dynamic systems and the cutter tool holder are partial and interconnected, it is possible to influence the carriage dominant systems nature oscillations by the tool holder’s oscillations. Because of this, it becomes an urgent issue to conduct theoretical studies of the elastic parameters of tool holders with an oriented center of rigidity, which are conducted at the Department of Machine Design [6], NTUU “Igor Sikorsky Kyiv Polytechnic Institute”. To carry out such research, a multi-mass mathematical model of the dynamic system "carriage - cutter holder - cutting process - workpiece" was built, taking into account the frequency characteristics of the forming units of the machine tools mod. 1K62, the physical properties of the workpieces material and the geometry of the cutting tool. Based on the results of dynamic system modeling, it was found that the use of a cutter holder with an oriented center of rigidity significantly increases the vibration resistance of processing compared to the use of a basic cutter holder.

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