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ELECTROMAGNENIC FIELD FOR A SYSTEM OF NONRELATIVIVISTIC CHARGES AT FAR DISTANCES

ЕЛЕКТРОМАГНІТНЕ ПОЛЕ ДЛЯ СИСТЕМИ НЕРЕЛЯТИВІСТСЬКИХ ЗАРЯДІВ НА ДАЛЕКИХ ВІДСТАНЯХ

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Abstract *In a nonstationary system of nonrelativistic charges, a change in the dipole moment of the whole system forms a dipole radiation, and a change of the monopole moment or total charge of the system creates at large distances a potential component of the electric field strength inversely proportional to the first degree of distance* $\vec{E}^{p}_{\psi}(R_0,t)$.

The source of the potential electric field at far distances or in the wave zone is a nonstationary potential energy. It's determined by the distance between charges or by the density of charges. The density of charges is the monopole moment of a unit volume, the change of which in time creates the strength of the potential electric field at far distances. The Eq. $\vec{E}^{p}_{\psi}(R_0,t)$ *is a wave process that propagates at a finite velocity. This leads to a retardation effect responsible for the creation of a potential electric field in the wave zone. TheEq.* $\vec{E}^{p}_{\psi}(R_0,t)$ *is a solution to the Dalembert wave equation, which includes explicit and implicit distance dependences. The latter determines the retardation effect.*

Using asymptotic expressions describing the retardation scalar and vector potentials of the system of nonrelativistic charges, the expansions on a small parameter of the electromagnetic field strengths in the form of retardation solutions are obtained, in which the retardation on the system size can be neglected in comparison with the retardation in the wave zone and the characteristic time of change of the charge and current densities of the system.

The assumption is used that at far distances the derivatives on the spatial variables between the source and observation points can be replaced by the derivatives on the spatial variables between the origin and the observation point. The electric and magnetic field strengths are obtained in space-time and space-frequency representations.

The retarded scalar and vector potentials determine the potential and dynamic components of electric energy, respectively, for unit charge and current densities at the observation point. Similarly, the potential and dynamical electric field strengths determine the force actions of the potential and dynamical actions, respectively, on the unit charge and current densities at the observation point.

Keywords: *the accelerating potential electric field, nonstationarymonopole moment, electric dipole moment, nonstationary electrodynamics, retarded scalar and vector potentials, electric*

displacement vector, displacement current, potential electric field strength ,wave zone, asymptotic expansion, small parameter, nonstationary charge density, charge density or monopole moment of unit volume, the characteristic time of change of the charge and current densities, antisymmetric current density.

1.Introduction

In the physics of charged particle beams, of essential importance are the collective effects. They manifest themselves through macroscopic quantities such as the charge density $\rho(\vec{r},t)$ and the current density $\vec{j}(\vec{r},t)$.

The electromagnetic field sources $\rho(\vec{r},t)$ and $\vec{j}(\vec{r},t)$ under consideration are distributed in the finite space region of volume V. The charge density for a charge element de occupying a volume dV is defined by the formula $de = \rho(\vec{r},t) \cdot dV$, where dV is a physically small volume, i.e., the volume, which is small, compared to the volume of the system, but is large as compared to the individual point charges in the form of electrons and ions.

The space-time variation of the dipole moment $K_1(\vec{r},t)$ of the charge system has been the subject of a large number of publications, while the number of publications dealing with changes in the monopole moment of the charge system $K_0(\vec{r},t)$, is very limited.. The monopole moment of the charge system is understood as the total charge of the system [1]. The retardation potentials, generated by the sources $\rho(\vec{r},t)$ and $\vec{j}(\vec{r},t)$, have been derived in [2] for the whole unlimited space.

In the monopole and dipole approximations the potentials for the system of nonrelativistic charges at far distances are obtained in [3]. In neglecting the retardation on the size of the system in the monopole approximation the scalar potential is found in [3] The latter is the zero approximation. The scalar and vector potentials in the dipole approximation are the first and zero approximations. The last pair satisfies the Lorentz gauge and forms the dipole radiation.

The existence of the potential electric field strength component, being inversely proportional to the first degree distance, was apparently first established in [4] (p.236-237). It was noted that reason for the occurrence of that sort of dependence resided in the retardation effect. At the harmonic time dependence, this component of the potential electric field is directed along the wave vector.

The potential electric field strength component, inversely proportional to the first degree of the distance between the source and the observation points at arbitrary distances, has been found in [5]. It is directed along the radius, describes an unsteady wave process in the general case, and is caused by charge density variations with time in the source region. This component accounts for the retardation effect at each point of the source. The spatial singularity is under the sign of the integral.

For a volume source, the potential component of the electric potential field strength, inversely proportional to the first degree of distance and directed along the radius drawn from the source point to the observation point, has been derived in [6] (Eq. 4.36). It was caused by the change of the volume charge density with time in the source region.

Furthermore, for a surface source, the unsteady scattered potential electric field strength, which is inversely proportional to the first degree of distance and is directed along the radius drawn from the source point to the observation point, has been also obtained in [6] (Eq. 4.42). It was due to the change of the surface charge density with time in the source region.

As a single electron moving along the normal to a perfectly conducting half-space stops abruptly, there arises the transient radiation comprising a longitudinal component of the electric field strength [7]. The longitudinal component is directed along the radius in the spherical coordinate system. The model of transient slowing-down radiation with application of the method of images was used.

2. Electromagnetic field

2.1. Retardation potentials

2.1.1 Space and time representation

Let us consider the electromagnetic field of a system of the nonrelativistic charges at distances large compared to the dimensions of the system. We put the origin - point O - inside the system (Figure1). In the figure, a point O' is a source point; a point P is an observation point; a vector $\vec{r}$ is a radius vector drawn from the origin to the source point; a vector $\vec{R}_0$ is a radius vector drawn from the origin to the observation point, $\vec{R}_0$ is a constant; a vector $\vec{R}$ is a vector drawn from the source point to the observation point; an angle φ is the angle between the vectors $\vec{r}$ and $\vec{R}_0$

In the approximation of far distances it can be written that

$$\left|\vec{R}\right| = \left|\vec{R}_0 - \vec{r}\right| = \sqrt{1 - 2\frac{r}{R_0}\cos\varphi + \frac{r^2}{R_0^2}} \approx$$

$$\approx R_0\sqrt{1 - 2\frac{1}{R_0}(\vec{r}\cdot\vec{n})} \approx R_0 - (\vec{n}\cdot\vec{r}) \quad , \tag{1}$$

$$t = \tau + \frac{R}{c} \approx \tau + \frac{R_0}{c} - \frac{(\vec{n}\cdot\vec{r})}{c} \quad , \tag{2}$$

where

$$r \ll R_0 , \tag{3}$$

$$\vec{n} = \vec{R}_0 / R_0 , \tag{4}$$

$$\vec{r} = \{ \ x, y, z \ \} , \tag{5}$$

$$\vec{R}_0 = \{ \ x_0, y_0, z_0 \ \} . \tag{6}$$

Taking into account Eq.(2), the charge and current densities can be written as

$$\rho(\vec{r},\tau) \approx \rho(\vec{r}, t - R_0/c + \frac{(\vec{n}\cdot\vec{r})}{c}) , \tag{7}$$

$$\vec{j}(\vec{r},\tau) \approx \vec{j}(\vec{r}, t - R_0/c + \frac{(\vec{n}\cdot\vec{r})}{c}) , \tag{8}$$

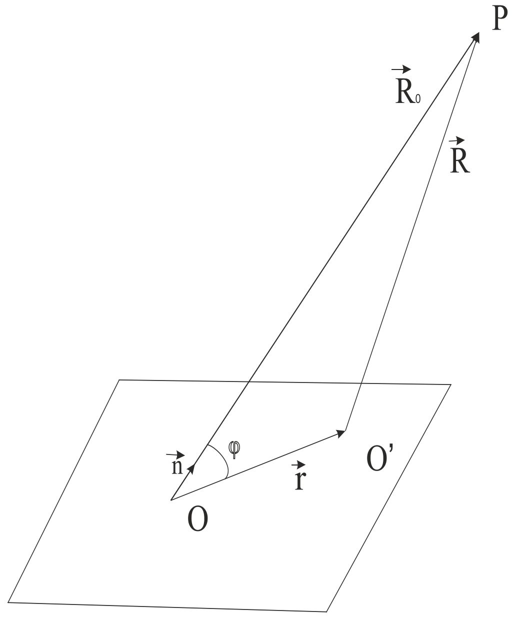

Figure1. Spatial coordinates in the source region and observation region.

In the asymptotic approximation at constant R_0, by virtue of the retardation effect the charge and current densities, being the functions depend on the spatial x, y, z and time τ coordinates in the source region transform into the functions of the spatial coordinates x, y, z in the source region and the time coordinate t at the observation point.

When decomposing by a small parameter

$$\eta = \frac{(\vec{n} \cdot \vec{r})}{c} \tag{9}$$

asymptotic expressions for the charge and current densities in the right parts Eq.(7) andEq. (8) we obtain

$$\rho(\vec{r},t) \approx \rho(\vec{r},t - R_0/c) + \frac{\partial \rho}{\partial t}\eta + \frac{\partial^2 \rho}{\partial t^2}\eta^2 + ..,\tag{10}$$

$$\vec{j}(\vec{r},t) \approx \vec{j}(\vec{r},t - R_0/c) + \frac{\partial \vec{j}}{\partial t}\eta + \frac{\partial^2 \vec{j}}{\partial t^2}\eta^2 + ...\tag{11}$$

The smallness parameter is a dimensional quantity equal to the retardation on the size of the nonrelativistic charge system.

An important quantity. requiring consideration at asymptotic decomposition, is the characteristic time of change of charge and current densities [3]

$$\tau \sim \frac{r_{\max}}{v_e}, \tag{12}$$

where $r_{\max}$ is the maximum system size and v_e is charge velocity, respectively. In this case

$$\frac{\tau}{\eta} \sim \frac{c}{v_e}, \tag{13}$$

i.e. for the system of nonrelativistic charges the characteristic time of change of charge and current densities essentially exceeds the retardation on the system size.

Taking into account Eq. (10) and Eq. (11) we find asymptotic expressions of retarded scalar and vector potentials for the system of nonrelativistic charges at far distances in the form of expansion by a small parameter

$$\psi(R,t) \approx \frac{1}{4\pi\varepsilon_0 R_0}\int_V dV\{\rho(\vec r, t-\frac{R_0}{c})\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} +$$
$$+\frac{1}{4\pi\varepsilon_0 R_0}\int_V dV\{\frac{\partial\rho}{\partial\tau}\cdot\frac{(\vec n\cdot\vec r)}{c}\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} +$$
$$+(1/2)(1/4\pi\varepsilon_0 R_0)\int_V dV\{\frac{\partial^2\rho}{\partial\tau^2}\cdot((\vec n\cdot\vec r)^2/c^2)\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} +\ldots \tag{14}$$

$$\vec A(R,t)\approx\frac{1}{4\pi c^2\varepsilon_0 R_0}\int_V dV\{\vec j(\vec r,t-R_0/c)\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} +$$
$$+(1/4\pi c^2\varepsilon_0 R_0)\int_V dV\{\frac{\partial\vec j}{\partial\tau}\frac{(\vec n\cdot\vec r)}{c}\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} +$$
$$+.(1/4\pi c^2\varepsilon_0 R_0)\int_V dV\{\frac{\partial^2\vec j}{\partial\tau^2}\frac{(\vec n\cdot\vec r)^2}{c^2}\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} +..$$

$$\vec A(R,t)\approx\frac{1}{4\pi c^2\varepsilon_0 R_0}\int_V dV\{\vec j(\vec r,t-R_0/c)\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} +$$
$$+(1/4\pi c^2\varepsilon_0 R_0)\int_V dV\{\frac{\partial\vec j}{\partial\tau}\frac{(\vec n\cdot\vec r)}{c}\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} +$$
$$+.(1/4\pi c^2\varepsilon_0 R_0)\int_V dV\{\frac{\partial^2\vec j}{\partial\tau^2}\frac{(\vec n\cdot\vec r)^2}{c^2}\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} +.. \tag{15}$$

In the first terms of the expansion in Eq.(14) and Eq.(15) we have neglected the retardation on the size of the system.

The first term in the expression for $\psi(R_0,t)$ is the total charge of the system $Q(t)$ divided by the distance to the observation point.

$$\psi_0(R_0,t)\approx(1/R_0)\{\int_V dV\rho(r,t-R_0/c,r)\}_{\tau\approx t-\frac{R_0}{c}+\frac{(\vec n\cdot\vec r)}{c}} = Q(t)/R_0\,. \tag{16}$$

The second term in Eq.(14) $\psi_1(t,R_0)$ is the scalar potential in the first or dipole approximation [3].

At far distances from the charge system

$$r \ll R_0 \tag{17}$$

and according to Eq. (14)

$$\psi_1(R_0,t)\ll\psi_0(R_0,t) \tag{18}$$

In an electrically charged system at far distances, the monopole moment makes a determining contribution to the scalar potential. The dipole moment and higher order moments can be neglected.

2.1.2 Space-frequency representation

The space-frequency representation of the retarded potentials is obtained using the Fourier transformation.

In the asymptotic approximation, the spectral component of the scalar potential over the observation time t [8], in view of Eq. (14), has the form

$$\psi^{t}(\omega\ ;\ R_0) \approx \frac{1}{4\pi\varepsilon_0 R_0}\int_V dV \cdot \int_{-\infty}^{+\infty} dt \cdot e^{i\omega t}\rho(t - R_0/c) +$$

$$+\frac{1}{4\pi\varepsilon_0 R_0}\int_V dV \int_{-\infty}^{+\infty} dt \cdot e^{i\omega t}\frac{\partial\rho}{\partial t}\cdot\frac{(\vec{n}\cdot\vec{r})}{c} +$$

$$+\frac{1}{2}\frac{1}{4\pi\varepsilon_0 R_0}\int_V dV \int_{-\infty}^{+\infty} dt \cdot e^{i\omega t}\cdot\frac{\partial^2\rho}{\partial t^2}\frac{(\vec{n}\cdot\vec{r})^2}{c^2} + \ldots \tag{19}$$

Let us first integrate over the retardation time τ. We reduce the Fourier transformation at the observation time t to the Fourier transform at the retardation time τ [8].

We multiply and divide the first term in Eq.(19) by $e^{-i\omega[\frac{R_0}{c}]}$, and the other terms are multiplied and divided by $e^{-i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}$

$$\psi^{t}(\omega\ ;\ R_0) \approx \frac{1}{4\pi\varepsilon_0 R_0}e^{i\omega[\frac{R_0}{c}]}\int_V dV \cdot \int_{-\infty}^{+\infty} dt \cdot e^{i\omega[t-R_0/c]}\rho(t - R_0/c, \vec{r}) +$$

$$+\frac{1}{4\pi\varepsilon_0 R_0}\int_V dV \cdot e^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty} e^{i\omega[t-R_0/c+\frac{(\vec{n}\cdot\vec{r})}{c}]}\cdot dt\cdot\frac{\partial\rho}{\partial\tau}\cdot\frac{\partial\tau}{\partial t}\frac{(\vec{n}\cdot\vec{r})}{c} +$$

$$+\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0 R_0}\int_V dVe^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty} e^{[t-R_0/c+\frac{(\vec{n}\cdot\vec{r})}{c})]}dt\frac{\partial^2\rho}{\partial\tau^2}\cdot(\frac{\partial\tau}{\partial t})^2\frac{(\vec{n}\cdot\vec{r})^2}{c^2} + \ldots \tag{20}$$

The Eq. (20) can be rewritten as

$$\psi^{t}(\omega\ ;\ R_0) \approx \frac{1}{4\pi\varepsilon_0 R_0}e^{i\omega[\frac{R_0}{c}]}\int_V dV \cdot \int_{-\infty}^{+\infty} d\tau_0 \cdot e^{i\omega\tau_0}\rho(\tau_0, \vec{r}) + \quad ,$$

$$+\frac{1}{4\pi\varepsilon_0 R_0}\int_V dV \cdot e^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty} e^{i\omega\tau}\cdot d\tau\cdot\frac{\partial\rho}{\partial\tau}\cdot\frac{\partial\tau}{\partial t}\frac{(\vec{n}\cdot\vec{r})}{c} +$$

$$+\frac{1}{2}\frac{1}{4\pi\varepsilon_0 R_0}\int_V dVe^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty} e^{i\omega\tau}d\tau\frac{\partial^2\rho}{\partial\tau^2}(\frac{\partial\tau}{\partial t})^2\frac{(\vec{n}\cdot\vec{r})^2}{c^2} + \ldots \tag{21}$$

where

$$\tau_0 = (t - R_0/c) \quad , \tag{22}$$

$$\tau = (t - \frac{R_0}{c} + \frac{(\vec{n}\cdot\vec{r})}{c}) . \tag{23}$$

Thus τ_0 and τ denote the retardation time for the cases with neglecting and with taking into account, respectively, the retardation on the size of the charge system.

The Equation (21) takes the form

$$\psi'(R_0,\omega) \approx \frac{1}{4\pi\varepsilon_0 R_0} e^{i\omega\frac{R_0}{c}} \int_V dV \cdot \rho(r,\omega) +$$

$$+\frac{1}{4\pi\varepsilon_0 R_0}\int_V dV \cdot e^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty} d\tau \cdot e^{i\omega\tau} \cdot \frac{\partial\rho}{\partial\tau}\cdot\frac{\partial\tau}{\partial t}\frac{(\vec{n}\cdot\vec{r})}{c} +$$

$$+\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0 R_0}\int_V dV e^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty} d\tau \cdot e^{i\omega\tau}\cdot\frac{\partial^2\rho}{\partial\tau^2}(\frac{\partial\tau}{\partial t})^2\frac{(\vec{n}\cdot\vec{r})^2}{c^2} + \ldots \tag{24}$$

or

$$\psi'(R_0,\omega) \approx \frac{1}{4\pi\varepsilon_0 R_0} e^{i\omega\frac{R_0}{c}} Q(\omega) + ,$$

$$+\frac{1}{4\pi\varepsilon_0 R_0}\int_V dV e^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty} e^{i\omega\tau} d\tau \frac{\partial\rho}{\partial\tau}\cdot\frac{\partial\tau}{\partial t}\cdot\frac{(\vec{n}\cdot\vec{r})}{c} +$$

$$+\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0 R_0}\int_V dV e^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty} e^{i\omega\tau} d\tau \frac{\partial^2\rho}{\partial\tau^2}(\frac{\partial\tau}{\partial t})^2\frac{(\vec{n}\cdot\vec{r})^2}{c^2} + \ldots \tag{25}$$

where

$$\rho(\omega) = \int_{-\infty}^{+\infty} e^{i\omega\tau 0} \cdot d\tau_0 \cdot \rho(\tau_0) . \tag{26}$$

is the Fourier image of the charge density of the system, and

$$Q(\omega) = \int_V dV \cdot \rho(r,\omega) \tag{27}$$

is the spectral density of the monopole moment or the total charge of the system

$$\psi'_0(\omega; R_0) \approx \frac{1}{4\pi\varepsilon_0 R_0} e^{i\omega\frac{R_0}{c}} Q(\omega) \tag{28}$$

describes the spectral density of the scalar potential in the form of the amplitude of a harmonic divergent spherical wave. The harmonic wave is produced by the spectral density of the effective charge $Q(\omega)$ located at the origin of coordinates. The expression $\psi'_0(\omega; R_0)$ does not take into account the phase change on the spatial inhomogeneity. We obtain the harmonic wave by means of multiplying Eq.(28) by $e^{-i\omega t}$,

$$\psi'_0(R_0,\omega,t) \approx \frac{1}{4\pi\varepsilon_0 R_0} e^{i\omega[-t+\frac{R_0}{c}]} Q(\omega) \tag{29}$$

With consideration for Eq. (11) in the asymptotic approximation, the spectral component of the vector potential over the observation time t [8] is obtained similarly to Eq. (21):

$$\vec{A}^{\,t}(R_0,\omega)\approx\frac{1}{4\pi c^2\varepsilon_0 R_0}e^{i\omega\frac{R_0}{c}}\int_V dV\cdot\vec{j}(\omega,r)+$$
$$\frac{1}{4\pi c^2\varepsilon_0 R_0}\int_V dV\cdot e^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty}d\tau\cdot e^{i\omega\tau}\frac{\partial\vec{j}}{\partial\tau}\cdot\frac{\partial\tau}{\partial t}\cdot\frac{(\vec{n}\cdot\vec{r})}{c}+.. \quad (30)$$

or

$$\vec{A}^{\,t}(R_0,\omega)\approx\frac{1}{4\pi c^2\varepsilon_0 R_0}e^{i\omega\frac{R_0}{c}}\vec{J}(\omega,r)+$$
$$+\frac{1}{4\pi c^2\varepsilon_0 R_0}\int_V dV\cdot e^{i\omega[\frac{R_0}{c}-\frac{(\vec{n}\cdot\vec{r})}{c}]}\int_{-\infty}^{+\infty}d\tau\cdot e^{i\omega\tau}\frac{\partial\vec{j}}{\partial\tau}\cdot\frac{\partial\tau}{\partial t}\cdot\frac{(\vec{n}\cdot\vec{r})}{c}+.. , \quad (31)$$

where

$$\vec{j}(\omega)=\int_{-\infty}^{+\infty}e^{i\omega\tau_0}\cdot d\tau_0\cdot\vec{j}(\tau_0) \quad (32)$$

is Fourier image of the current density of the system, and

$$\vec{J}(\omega)=\int_V dV\cdot\vec{j}(\omega,r) \quad (33)$$

is the spectral density of the effective current of the system.

The first summand in Eq. (31

$$\vec{A}_0^{\,t}(\omega;R_0)\approx\frac{1}{4\pi c^2\varepsilon_0 R_0}e^{i\omega\frac{R_0}{c}}\vec{J}(\omega,r) \quad (34)$$

in the zero approximation describes the spectral density of the vector potential as the amplitude of a harmonic divergent spherical wave. The harmonic wave is created by the spectral density of the effective current $\vec{J}(\omega)$ located at the origin. We obtain the harmonic wave by multiplying $e^{-i\omega t}$ bythe Eq. (31).

$$\vec{A}_0(\omega;t,R_0)\approx\frac{1}{4\pi c^2\varepsilon_0 R_0}e^{i\omega[\frac{R_0}{c}-t]}\vec{J}(\omega,r) \quad (35)$$

The expression $\vec{A}_0(\omega;R_0)$ does not account for phase runup on spatial inhomogeneity.

2.2. Electromagnetic field strengths

2.2.1.Space and time representation

In the zero-point approximation, the electric $\vec{E}(\vec{R}_0,t)$ and magnetic $\vec{H}(\vec{R}_0,t)$ field strengths are dependent on the retarded scalar $\psi(\vec{R}_0,t)$ and vector $\vec{A}(R_0,t)$ potentials, and are determined by the relations [9]] (p.432)

$$\vec{E}(x_0,y_0,z_0;t)=-grad_{R_0}\psi(x_0,y_0,z_0;t)-\frac{\partial}{\partial t}\vec{A}(x_0,y_0,z_0;t) , \quad (36)$$

$$\vec{H}(x_0, y_0, z_0; t) = \frac{1}{\mu_0} rot_{R_0} \vec{A}(x_0, y_0, z_0; t) \tag{37}$$

In Eq. (36) the term

$$\vec{E}^{\,p}(x_0, y_0, z_0; t) = -grad_{R_0} \psi(x_0, y_0, z_0; t) \tag{38}$$

is the potential component of the electric field strength, and the term

$$\vec{E}^{\,r}(x_0, y_0, z_0; t) = -\frac{\partial \vec{A}(x_0, y_0, z_0; t)}{\partial t} \tag{39}$$

is the dynamic component of the electric field strength.

Up to order terms $(\frac{r}{R_0})^2$, taking into account Eqs. (14), (38), we have for

$$\vec{E}^{\,p}(x_0, y_0, z_0; t) = -grad_{R_0} \psi(x_0, y_0, z_0; t) \approx -grad_{R_0} \psi_0(x_0, y_0, z_0; t) \tag{40}$$

In an electrically charged system at far distances, the spatial derivative of the monopole moment makes a determining contribution to the potential component of the electric field strength. The derivative of the dipolemoment and moments of higher orders can be neglected.

With precision to terms proportional to $1/R_0^2$

$$-grad_{R_0} \psi_0(x_0, y_0, z_0; t) = -\frac{\partial}{\partial x_0} \psi_0(x_0, y_0, z_0; t)\vec{i} -$$

$$\frac{\partial}{\partial y_0} \psi_0(x_0, y_0, z_0; t)\vec{j} - \frac{\partial}{\partial z_0} \psi_0(x_0, y_0, z_0; t)\vec{k} \approx$$

$$\approx -\frac{1}{R_0} \int_V dV \{\frac{\partial \rho(\tau)}{\partial \tau} \cdot \frac{\partial \tau}{\partial x_0}\}_{\tau \approx t - \frac{R_0}{c}} dxdydz\vec{i} + ... \tag{41}$$

In this case

$$t - \tau = \frac{|\vec{R}_0 - \vec{r}|}{c} \tag{42}$$

In the asymptotic approximation

$$\tau \approx t - R_0/c + \frac{(\vec{n} \cdot \vec{r})}{c} \tag{43}$$

With accuracy up to the terms proportional to x/R_0 we have

$$\frac{\partial \tau}{\partial x_0} \approx -\frac{1}{c}\frac{\partial R_0}{\partial x_0} \approx -\frac{1}{c}\frac{1}{2}\frac{2x_0}{R_0} \approx -\frac{1}{c}\cos(\vec{x}_0, \vec{R}_0) \tag{44}$$

Taking into account Eqs.(44), (41) lead to the following expression

$$\vec{E}^{p}_{\psi}(R_0,t) \approx -grad_{R_0}\psi_0(t,\vec{R}_0) \approx (1/4\pi\varepsilon_0 R_0)\cdot$$

$$\cdot\int_V \{\frac{\partial\rho(\tau;\vec{r}))}{\partial\tau}\cdot\frac{\cos(\vec{x}_0,\vec{R}_0)}{c})\}_{\tau\approx t-\frac{R}{c}+\frac{(\vec{n}\cdot\vec{r})}{c}} dV\cdot\vec{i}_0 + \ldots \approx$$

$$\approx \frac{1}{4\pi\varepsilon_0 cR_0}\int_V \frac{\partial\rho(\tau;\vec{r}))}{\partial\tau}[\cos(\vec{x}_0,\vec{R}_0)\vec{i}_0 + \cos(\vec{y}_0,\vec{R}_0)\vec{j}_0 +$$

$$\cos(\vec{z}_0,\vec{R}_0)\vec{k}_0]\}_{\tau\approx t-\frac{R0}{c}+\frac{(\vec{n}\cdot\vec{r})}{c}} dV \approx$$

$$\approx 1/4\pi\varepsilon_0 cR_0 \int_V \{\frac{\partial\rho(\tau;\vec{r}))}{\partial\tau}\cdot\vec{n}(\vec{R}_0)\}_{\tau\approx t-\frac{R0}{c}+\frac{(\vec{n}\cdot\vec{r})}{c}} dV \approx$$

$$\approx \vec{n}(x_0,y_0,z_0)/4\pi\varepsilon_0 cR_0 \int_V \{\frac{\partial\rho(\tau;\vec{r}))}{\partial\tau}\}_{\tau\approx t-\frac{R0}{c}+\frac{(\vec{n}\cdot\vec{r})}{c}} dV \quad , \qquad (45)$$

where $\vec{n}(x_0,y_0,z_0)$ is the unit vector along the radius vector $\vec{R}_0$, and $\vec{i}_0,\vec{j}_0,\vec{k}_0$ are the unit vectors along the Ox, Oy, Oz axes, respectively.

The formula (45) with the accuracy up to the proportional terms $1/R_0^2$ determines at far distances or in the wave zone the potential component of the electric field strength, defined by the scalar potential.

According to Eqs. (14), (16) and (45), the scalar potential $\psi(R_0,t)$ and the potential component of the electric field strength $\vec{E}^{p}_{\psi}(R_0,t)$ in the wave zone are inversely proportional to the first degree of the distance from the source to the observation point.

In this case, $\psi(R_0,t)$ in the wave zone is determined by the change of the electric monopole moment $K_0(\tau)$, equal (within the allocated source volume V) to the charge at the retarded moment of time τ.

$$K_0(\tau) = Q(V,\tau) = \int_V \rho(x,y,z;\tau)dV \qquad (46),$$

And $\vec{E}^{p}_{\psi}(R_0,t)$ in the wave zone is caused by the change in the time derivative of the electric monopole moment $\frac{\partial K_0(\tau)}{\partial\tau}$, equal (within the allocated source volume V) to the velocity of change for the charge $\frac{\partial Q(\tau,V)}{\partial\tau}$ at the retarded moment of time τ

$$\frac{\partial K_0(\tau)}{\partial\tau} = \int_V \frac{\partial\rho(x,y,z;\tau)}{\partial\tau}dV. \qquad (47)$$

or

$$\frac{\partial K_0(\tau)}{\partial\tau} = \int_V -div\vec{j}(x,y,z;\tau)\cdot dV. \qquad (48)$$

The derivative $\dfrac{\partial K_0(\tau)}{\partial \tau}$ can also be represented as a flux of charge through a closed surface S bounding the given volume V

$$\frac{\partial K_0(\tau)}{\partial \tau} = \oint_S -(\vec{j}(\vec{r};\tau)\cdot d\vec{S}) \tag{49}$$

The potential component of the electric field strength $\vec{E}_\psi^p(R_0,t)$ in the wave zone is directed along the normal drawn from the origin of coordinates in the source region to the observation point, and its distribution is isotropic. In the SI system, it corresponds to the second summand in (6.55) [5] at large R_0 values.

Taking into account the expression (15) for the vector potential $\vec{A}(R_0,t)$, we obtain

$$-\frac{\partial \vec{A}(t,R_0(x_0,y_0,z_0))}{\partial t} = -\frac{1}{4\pi c^2 \varepsilon_0 R_0}\int_V \{\frac{\partial \vec{j}(\tau(t,\vec{r};\vec{R}_0))}{\partial \tau}\cdot$$
$$\cdot\frac{\partial \tau}{\partial t}\}_{\tau=t-\frac{|\vec{R}_0-\vec{r}(x,y,z)|}{c}} dxdydz \tag{50}$$

According to Eq.(2),

$$\frac{\partial \tau}{\partial t} \approx \frac{1}{[1-\frac{1}{c}(\vec{n}\cdot\vec{v}(\vec{r}))]}, \tag{51}$$

where $\vec{v}(\vec{r})$ is the velocity of the moving charge at the point $O'(\vec{r})$.

According to Eqs.(50), (51), we obtain

$$\vec{E}_A^r(R_0,t) \approx \frac{(-1)}{4\pi c^2 \varepsilon_0 R_0}\int_V dxdydz\cdot$$
$$\cdot\{\frac{\partial \vec{j}(R_0.\tau(t,r;R_0))}{\partial \tau}\frac{1}{[1-\frac{1}{c}(\vec{n}\cdot\vec{v}(x,y,z))]}\}_{\tau\approx t-\frac{R_0}{c}} \tag{52}$$

The formula (52) defines the dynamic component of the electric field strength $\vec{E}_A^r(R_0,t)$ in the wave zone. It is proportional to the second derivative with respect to the retarded time from the dipole moment of the selected volume V in the source region, is directed along this derivative, and is inversely proportional to the retardation factor.

The dynamic component of the electric field strength $\vec{E}_A^r(R_0,t)$ in the wave zone corresponds to the third summand in (6.55) [5] at large R_0.

Combining Eq.(45) and Eq. (52), we obtain the asymptotic expression for the resulting electric field strength $\vec{E}_{\psi,A}(R_0,t)$ of the system of nonrelativistic charges in vacuum at far distances or in the wave zone:

$$\vec{E}_{\psi,A}(R_0,t)=\vec{E}^{p}_{\psi}(R_0,t)+\vec{E}^{r}_{A}(R_0,t)\approx(\vec{n}(R_0)/4\pi\varepsilon_0 cR_0)$$
$$\int_V\{\frac{\partial\rho(r;\tau))}{\partial\tau}\}_{\tau\approx t-\frac{R_0}{c}}dV-\frac{1}{4\pi c^2\varepsilon_0 R_0}\cdot$$
$$\cdot\int_V\{\frac{\partial\vec{j}(\tau(t,r;R_0))}{\partial\tau}\frac{1}{[1-\frac{1}{c}(\vec{n}\cdot\vec{v}(r))]}\}_{\tau\approx t-\frac{R_0}{c}}dV \quad (53)$$

In relation (53), we neglected the retardation on the size of the system of nonrelativistic charges in comparison with the retardation between the system and the observation point.

The potential component of the electric field strength $\vec{E}^{p}_{\psi}(R_0,t)$ in the wave zone is directed along the normal drawn from the origin of coordinates in the source region to the observation point, and its distribution is isotropic. In the SI system, it corresponds to the second summand in (6.55) [5] at large R_0 .

For the system of nonrelativistic charges in vacuum at far distances or in the wave zone, the electric field strength has a monopole-dipole character and consists of potential and dynamic components.

The magnetic field strength, according to Eq.(37) and Eq. (15) with an accuracy of up to the terms proportional to $1/R_0^2$ has the form

$$\vec{H}(\vec{R}_0,t)=\frac{1}{4\pi R_0}\int_V dV\{\begin{vmatrix}\vec{i}_0 & \vec{j}_0 & \vec{k}_0\\ \frac{\partial}{\partial x_0} & \frac{\partial}{\partial y_0} & \frac{\partial}{\partial z_0}\\ j_x & j_y & j_z\end{vmatrix}\}_{\tau=t-\frac{|\vec{R}_0-\vec{r}|}{c}}\approx \quad (54)$$

Similar to Eq.(44), we have

$$\frac{\partial\tau}{\partial y_0}\approx-\frac{1}{c}\cos(\vec{y}_0,\vec{R}_0), \quad (55)$$

$$\frac{\partial\tau}{\partial z_0}\approx-\frac{1}{c}\cos(\vec{z}_0,\vec{R}_0). \quad (56).$$

Taking into account Eqs. (54) - (56), (44), we have

$$H_x(R_0,t)=1/4\pi R_0 c\int_V dV\{-\frac{\partial j_z(\tau)}{\partial\tau}\cos(\vec{y}_0,\vec{R}_0)+$$
$$+\frac{\partial j_y(\tau)}{\partial\tau}\cos(\vec{z}_0,\vec{R}_0)\}_{\tau=t-\frac{|\vec{R}_0-\vec{r}|}{c}} \quad (57).$$

$$H_y(R_0,t)=1/4\pi R_0 c\int_V dV\{-\frac{\partial j_x(\tau)}{\partial\tau}\cos(\vec{z}_0,\vec{R}_0)+$$
$$+\frac{\partial j_z(\tau)}{\partial\tau}\cos(\vec{x}_0,\vec{R}_0)\} \quad (58).$$

$$H_z(R_0,t) = 1/4\pi R_0 c \int_V dV\{-\frac{\partial j_y(\tau)}{\partial\tau}\cos(\vec{x}_0,\vec{R}_0) + $$
$$+\frac{\partial j_x(\tau)}{\partial\tau}\cos(\vec{y}_0,\vec{R}_0)\}_{\tau=t-\frac{|\vec{R}_0-\vec{r}|}{c}} \quad (59).$$

According to Eqs.(57)-(59) the magnetic field strength for a system of nonrelativistic charges in vacuum at far distances or in the wave zone is proportional to the second derivative on retarded time of the dipole moment of the source

The magnetic displacement vector is written in the form

$$\vec{B}(\vec{R}_0,t) \approx \frac{\mu_0}{4\pi R_0 c}\int_V dV\{\frac{\partial\vec{j}(\tau,r)}{\partial\tau}\times\vec{n}(x_0,y_0,z_0)\}_{\tau=t-\frac{|\vec{R}_0-\vec{r}|}{c}} \quad (60)$$

The magnetic displacement vector in the wave zone corresponds to the second summand in (6.56) [5] at large $\vec{R}_0$.

It follows from an equations (53) and (54) that the scalar multiplication of the vectors $\vec{E}_{\psi,A}(R_0,t)$ and $\vec{H}(R_0,t)$ is equal to zero. The vectors of the resulting electric field strength and magnetic field strength are mutually orthogonal at large distances.

2.2.2 Space frequency representation

We obtain the space-frequency representation of the electromagnetic field strength in terms of harmonically divergent spherical waves, using the space-frequency representation in the form of a harmonically divergent spherical wave of the scalar (Eq. (29)) and vector (Eq. (35)) potentials According to Eqs. (45), (29), we obtain

$$\vec{E}^t_\psi(R_0,\omega;t) \approx -grad_{R_0}\psi_0(R_0,\omega;t) \approx -grad_{R_0}[\frac{1}{4\pi\varepsilon_0 R_0}\cdot$$
$$\cdot e^{i\omega[\frac{R_0}{c}-t]}Q(\omega)] \approx -\frac{i\cdot e^{i\omega[\frac{R_0}{c}-t]}Q(\omega)}{4\pi\varepsilon_0 R_0}\cdot\vec{k}(\omega) \quad , \quad (61)$$

where

$$\vec{k}(\omega) = \{\ k_x(\omega), k_y(\omega), k_z(\omega)\ \} =$$
$$=\left\{\frac{\omega}{c}\cos(\vec{x}_0,\vec{R}_0), \frac{\omega}{c}\cos(\vec{y}_0,\vec{R}_0),\ \frac{\omega}{c}\cos(\vec{z}_0,\vec{R}_0)\right\} =$$
$$=\frac{\omega}{c}\vec{n}(\vec{R}_0) \quad (62)$$

is the wave vector along the normal line.

According to the space-frequency representation, Eq. (61) in the zero-point approximation defines the potential component of the electric field strength in the form of a harmonic divergent spherical wave in the wave zone. The harmonic wave amplitude is proportional to the spectral density of the total charge of the system or its monopole moment. The potential component is directed along the wave vector, i.e., the vectors $\vec{E}_{\psi}^{t}(R_0,\omega;t)$ and $\vec{k}(\omega)$ are parallel

According to equations (35), (39), we have

$$\vec{E}_{A}^{t}(R_0,\omega;t)\approx\frac{i\omega}{4\pi c^2\varepsilon_0 R_0}e^{i\omega[\frac{R_0}{c}-t]}\vec{J}(\omega) \tag{63}$$

An equation (63) defines the dynamic component of the electric field strength in the space-frequency representation in the form of a harmonic divergent spherical wave in the wave zone. The angle between the vectors $\vec{E}_{A}^{t}(R_0,\omega;t)$ and $\vec{k}(\omega)$ is equal to the angle between the vectors $\vec{J}(\omega)$ and $\vec{k}(\omega)$.

Combining equations (61) and (63), we obtain the resultant electric field strength in the space-frequency representation as a sum of two harmonic divergent spherical waves in the wave zone

$$\vec{E}_{\psi,A}^{\,t}(R_0,\omega;t)=\vec{E}_{\psi}^{pt}(R_0,\omega;t)+\vec{E}_{A}^{rt}(R_0,\omega;t)\approx$$
$$\approx-\frac{i\cdot e^{i\omega[\frac{R_0}{c}-t]}Q(\omega)}{4\pi\varepsilon_0 R_0}\cdot\vec{k}(\omega)+\frac{i\omega}{4\pi c^2\varepsilon_0 R_0}e^{i\omega[\frac{R_0}{c}-t]}\vec{J}(\omega). \tag{64}$$

According to equations (35), (37), we obtain

$$\vec{H}(R_0,\omega;t)\approx\frac{i\omega}{4\pi c\ R_0}e^{i\omega[\frac{R_0}{c}-t]}\Big\{\vec{i}_0[-J_z(\omega)\cos(\vec{y}_0,\vec{R}_0)+$$
$$+J_y(\omega)\cos(\vec{z}_0,\vec{R}_0)]+$$
$$+\vec{j}_0[-J_x(\omega)\cos(\vec{z}_0,\vec{R}_0)+J_z(\omega)\cos(\vec{x}_0,\vec{R}_0)]+$$
$$+\vec{k}_0[J_x(\omega)\cos(\vec{y}_0,\vec{R}_0)-J_y(\omega)\cos(\vec{x}_0,\vec{R}_0)]\} \tag{65}$$

An equation (65) describes the magnetic field strength in the space-frequency representation in the wave zone as a harmonic divergent spherical wave.

3. CONCLUSIONS

It is shown that the system of nonrelativistic charges in vacuum at distances considerably exceeding its own dimensions forms a potential component of the vector potential.

It is also shown that the system of the nonrelativistic charges in vacuum at distances considerably exceeding its own dimensions forms a potential component of the electric field strength, inversely proportional to the first degree of the distance between the source and observation points.

Due to the nonstationarity of the system of non-relativistic charges, a chain arises: time variation of the monopole moment; free charge flux; displacement current.

Decompositions on a small parameter of electromagnetic field strengths in the space-time representation in the form of retarded solutions in which the charge density and current density change slowly during the time of propagation of the electromagnetic field along the system of charges are obtained.

The potential electric field strength at large distances is proportional to the derivative of the monopole moment or the charge flux through the boundary closed surface at the retarded moment of time.

When in the system of non-relativistic charges moving in vacuum, the distances between the charges change, the system forms the potential component of the electric field strength, inversely proportional to the first degree of at far distances.

The potential component of the electric field strength consists of two terms. The first and second terms are proportional to the gradient of the scalar potential and the time derivative for the potential part of the vector potential, respectively.

For the system of nonrelativistic charges in vacuum at far distances or in the wave zone, the electric field strength has a monopole-dipole character and consists of potential and dynamic components.

The asymptotic representation for the electromagnetic field strength is obtained in the form of a small-parameter expansion. The zero approximation describes in general the nonstationary wave process, neglecting the retardation on the system size.

The reason for this phenomenon is local: change of the charge density or antisymmetric component of the current density, and integral: change of the monopole momentum for the system of moving charges.

The strength of the potential electric field at large distances is proportional to the derivative of the monopole moment or the flux of charge through the boundary closed surface at the retarded moment of time.

In an nonstationary electrically charged system at large distances, the spatial derivative of the monopole moment makes a determining contribution to the potential component of the electric field strength. The derivative of the dipole moment and higher order moments can be neglected.

The spatial singularity between the source and observation points is taken out from under the sign of the integral.

The presence at large distances of a potential component comparable to the rotational component of the electric field strength is determined by the retardation effect. There is a connection between events or spatial and ttime coordinates at the source and observation points.

At distances comparable to the size of the charge system, there is a potential component of the electric field strength inversely proportional to the second degree of the distance between the source and observation

For the system of nonrelativistic charges in vacuum at far distances or in the wave zone, the electric field strength has a monopole-dipole character and consists of potential and dynamic components.

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