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DYNAMICS OF SOLITONS IN ADHESIVE COMPOSITE SYSTEMS

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Abstract. *This paper presents an analytical model for studying the dynamics of wave generation and propagation in composite adhesive structures. The composite structures are modeled as shear-deformed beams and investigated using first-order shear deformation theory. The propagation properties of high-frequency waves are studied. Hamilton's principle is used to obtain the equations of wave motion in adhesive composites. The layered structure of the composites is compared to a set of continuously distributed elastic elements characterized by shear deformation. The time-frequency transformation of the governing equations of motion is performed based on scaling Daubechies wavelets. A methodology for spectral transformation of the finite element approach has been developed to describe the relationship between nodal forces and displacements in the dynamic stiffness matrix. Results for spectral frequency analysis are obtained. Boundary conditions for the dispersion relation are formulated explicitly.*

Key words: *spectral wavelet analysis, wave propagation, composite structures, double beam system.*

Introduction.

The main advantages of reinforced composite structures include higher fatigue strength and longer fatigue life, low weight, the ability to join thin and dissimilar components, good sealing, low manufacturing cost, and good vibration and damping properties [1]. Currently, methods for studying reinforced composites based on ultrasonic waves are widely used [2]. Among such methods, one can single out studies of the physical and chemical properties of individual layers of composites. Detailed local analysis of mechanical stresses in the volumes of a composite structure can be carried out using experiments using ultrasonic waves, including acoustic emission, volume and directed (longitudinal, shear and Lamb) waves, nonlinear solitary waves, etc. Among the analytical methods, one can single out a one-dimensional semi-analytical finite element model for predicting dispersion curves and mode shapes of directed shear-horizontalization waves [3, 4].

It should be emphasized, however, that such waves propagate along multilayer plates made of anisotropic and viscoelastic materials (composite layers). The sensitivity of the technique to the sample thickness for the transfer function of a multilayer medium can be estimated based on the exact solution of the one-dimensional wave equation. Experiments involving the study of longitudinal and bending vibrations of parallel and identical cantilevers connected by overlapping using a viscoelastic binder allow us to determine the numerical values of both natural frequencies and loss factors.

The final results of such a technique include the possibility of obtaining a complete set of Euler–Bernoulli equations for motion and boundary conditions that regulate the dynamics of the system based on beam theory. The physical model of wave propagation in combination with experimental measurements allows to give a complete characteristic of the average concentration, local location and geometric factor of damage [5, 6].

Modeling of wave propagation in composites assumes the presence of non-periodic inhomogeneities of the location of local mechanical damages. The finite element method is the most popular numerical method for modeling wave propagation phenomena. It should be noted, however, that for accurate predictions using the finite element method, as a rule, it is necessary to have an indexing network consisting of at least several dozen elements that covers the wavelength. Fulfilment of this condition usually leads to a very large size of the system and huge computational costs for the analysis of wave propagation at high frequencies.

The spectral finite element characteristics that can be determined from the frequency domain finite element modeling procedure are quite convenient for wave propagation analysis. The used scale of the spectral finite element model is many orders of magnitude smaller than the finite element model and allows introducing a physically based concept of an effective spectral finite element. The frequency domain formulation of the spectral finite element provides a direct connection between the output and the input via the system transfer function (frequency response function). In addition, the use of computational methods shows that the spectral finite element has a very high computational efficiency since the nodal displacements are related to the nodal thrusts via a stiffness matrix that depends on the frequency and number of waves [7, 8]. In the spectral finite element, the mass distribution can be localized with a large degree of detail and, in addition, in this case, the theoretical model assumes the definition of an accurate elemental dynamic stiffness matrix.

Wavelet approximation of governing equations

The first model, which determined the shape of the composite structure, was represented as a free body of a double beam system (representing the overlap zone of a single lap joint). We denote the values of normal, shear and bending resulting stresses by the symbols N_i, Q_i and M_i, respectively, for each of the beams with the number $i = 1, 2$. The normal and shear forces of interaction between local areas of the reinforced composite are specified as the quantities p and s, measured as force per unit length. The transverse shear of the layer located parallel to the side surface of the composite specimen and in the direction of propagation of acoustic waves is assumed to be negligible, since its thickness is much smaller than the thickness of the entire specimen. The corresponding cross-sections are considered as fixed rectangles with an area of $A = 2h_i b$.

The displacement fields for axial and transverse shear in a beam can be defined as

$$u(x, y, t) = u_c(x, t) - y\varphi(x, t) \tag{1}$$

$$\upsilon(x, y, t) = \upsilon_c(x, t). \tag{2}$$

where u and v are, respectively, the axial and transverse displacements for a fixed region of the specimen;

u_c and v_c are the axial and transverse displacements of the beam along the supporting midplane (relative to the entire volume).

Most analytical models that describe the dynamics of the origin and propagation of local strain fields use general nonlinear strain-displacement relations. These relations are based on the assumption that only quasi-elastic strains exist in the sample under consideration. However, considering only small strains and displacements does not significantly limit the scope of analysis of the mechanical properties of the entire composite material. Linear strains associated with the above-mentioned displacement field are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_c}{\partial x} - y\frac{\partial \varphi}{\partial x}, \quad (3)$$

$$\gamma_{xx} = \frac{\partial \upsilon}{\partial x} + \frac{\partial \upsilon}{\partial y} = \frac{\partial \upsilon_c}{\partial x} - \varphi, \quad (4)$$

$$\varepsilon_{xx} = \varepsilon_{zz} = \gamma_{xz} = \gamma_{yz} = 0. \quad (5)$$

The sum of the strain energy and potential energy can be represented as the total potential energy. The magnitude of the strain energy is determined by internal and external mechanical forces. Denoting the volume of the composite sample by the symbol V, we can represent the virtual strain energy as the following expression

$$\delta U = \int_V \sigma_{ij}\delta\varepsilon_{ij}dV. \quad (6)$$

where δ is the variational operator;

σ_{ij} are stress terms.

Using the functional relationship between the values of mechanical stress σ_{ij} and tensors N_{ij}, M_{ij} and Q_{ij} for variation of U we can write

$$\delta U = -\int_0^L \left[\frac{\partial N_{xx}}{\partial x}\delta u_c - \frac{\partial M_{xx}}{\partial x}\delta\varphi + \frac{\partial Q_{xx}}{\partial x}\delta\upsilon_c + Q_{xx}\delta\varphi\right]. \quad (7)$$

Provided that the previously made assumptions are fulfilled, the variation of the virtual kinetic energy of local deformation regions can be represented as

$$\delta K = -\int_0^L \left[I_0\frac{\partial^2 u_c}{\partial t^2}\delta u_c - I_1\frac{\partial^2 u_c}{\partial t^2}\delta\varphi - I_1\frac{\partial^2 \varphi}{\partial t^2}\delta u_c + I_2\frac{\partial^2 \varphi}{\partial t^2}\delta\varphi + I_0\frac{\partial^2 \upsilon_c}{\partial t^2}\delta\upsilon_c\right], \quad (8)$$

where I_k (k = 0, 1, 2) are the inert terms that are matrix functions of the coordinates and geometric parameters of the composite sample.

It should be noted that within the framework of the numerical model of the wavelet transform using the finite element method, the inertial term I_1 will be equal to zero for the geometric location of points located in the midplane of the sample. According to Hamilton's principle, the functional relationship between the values of kinetic and total potential energy for a conservative elastic body can be written in the form of the Lagrange variational equation

$$\delta\int_{t_1}^{t_2}[K - (U + V)]dt = 0. \quad (9)$$

Assuming that the virtual displacements for times 1 and 2 are equal to zero, we write the coefficients for the Euler-Lagrange equation

$$\delta u_{1c} : \frac{\partial N_{xx1}}{\partial x} + s - I_0\frac{\partial^2 u_{1c}}{\partial t^2} + I_1\frac{\partial^2 \varphi_1}{\partial t^2} = 0, \quad (10)$$

$$\delta\upsilon_{1c}:\frac{\partial Q_{x1}}{\partial x}-p-I_0\frac{\partial^2\upsilon_{1c}}{\partial t^2}=0, \tag{11}$$

$$\delta\varphi_1:-\frac{\partial M_{xx1}}{\partial x}+Q_{x1}+h_1s+I_1\frac{\partial^2u_{1c}}{\partial t^2}-I_2\frac{\partial^2\varphi_1}{\partial t^2}=0. \tag{12}$$

Next, the resulting forces and moments given in the equation are related to the deformations of the reinforced composite through the governing equations. For this purpose, it is assumed that each layer is orthotropic with respect to the symmetry lines of its material and obeys Hooke's law. The deformation expressions given in the system of equations are then used for the finite difference method and the boundary conditions in terms of displacement.

The functional relationships for axial, axial bending and bending stiffness are defined as follows

$$A_{11}=b\sum_{k=1}^{N_p}\overline{Q}_{11}^{(k)}\left(y_{k+1}-y_k\right), \tag{13}$$

$$B_{11}=\frac{1}{2}b\sum_{k=1}^{N_p}\overline{Q}_{11}^{(k)}\left(y_{k+1}^2-y_k^2\right), \tag{14}$$

$$D_{11}=\frac{1}{3}b\sum_{k=1}^{N_p}\overline{Q}_{11}^{(k)}\left(y_{k+1}^3-y_k^3\right), \tag{15}$$

where $\overline{Q}_{11}^{(k)}$ represents the mechanical rigidity of the k-th layer of the reinforced composite;

N_p total number of layers for a fixed local sample volume.

The system of equations is written under the assumption of the presence of normal and shear forces for each deformation in the sample volume. These forces can be derived through the constitutive relations of the adhesive layer.

The constitutive relations for the adhesive layer are established using a two-parameter elastic foundation approach, where it is assumed that the adhesive layer consists of continuously distributed shear deformations.

For comparison, a second model was considered, in which a statistical approach was used to study the coupled longitudinal and bending vibrations of reinforced composites. The magnitudes of the interaction forces p and s in this case are described by a system of equations

$$s=k_s\left(u_{2c}-h_2\varphi_2-u_{1c}-h_1\varphi_1\right),\qquad k_s=\frac{G_a}{t_a}, \tag{16}$$

$$p=k_t\left(\upsilon_{1c}-\upsilon_{2c}\right),\qquad k_t=\frac{E_a}{t_a}, \tag{17}$$

where E_a, G_a are Young’s and shear moduli, respectively;

k_s, k_t are spring constants into X and Y directions.

It should be noted that the physical meaning of the terms k_s and k_t is that they can be represented as elastic constants in the X and Y directions. These constants are the key parameters in the system of partial differential equations for the double beam system. It should be noted that the above procedure can be used to formulate the discrete numerical analysis for other coupled structures (with a large number of elements), such as a lap joint. In addition, the governing differential equations written earlier are a system of coupled linear partial differential equations, which are difficult to solve exactly in the time domain for all boundary conditions. A way out of this situation is to use the spectral dynamic description of acoustic wave propagation. The solution for a given load and boundary conditions is obtained in the frequency domain, and the inverse wavelet transform is performed to obtain the results in the time domain.

The model assumes the specification of a system of partial differential equations to describe the wave dynamics in each layer separately. In the equations, the subscripts correspond to the layer number, counted from the side surface of the composite sample. After this, the transformed control equations for a fixed layer are written as

$$A_{11}\frac{d^2\hat{u}_{1c}}{dx^2} - B_{11}\frac{d^2\hat{\varphi}_1}{dx^2} + \hat{S} + I_0\gamma^2\hat{\varphi}_1 = 0, \quad (18)$$

$$\kappa A_{12}\left(\frac{d^2\hat{\upsilon}_{1c}}{dx^2} - \frac{d\hat{\varphi}_1}{dx}\right) - \hat{p} + I_0\gamma^2\hat{\upsilon}_{1c} = 0, \quad (19)$$

$$-B_{11}\frac{d^2\hat{u}_{1c}}{dx^2} + D_{11}\frac{d^2\hat{\varphi}_1}{dx^2} + A_{12}\left(\frac{d\hat{\upsilon}_{1c}}{dx} - \varphi_1\right) + h_1\hat{S} - I_1\gamma^2\hat{u}_{1c} + I_2\gamma^2\hat{\varphi}_1 = 0. \quad (20)$$

This system of equations is supplemented by boundary conditions for two adjacent layers

$$\hat{\bar{N}}_{xx1} = A_{11}\frac{d\hat{u}_{1c}}{dx} - B_{11}\frac{d\hat{\varphi}_1}{dx}, \qquad \hat{\bar{Q}}_{x1} = \kappa A_{12}\left(\frac{d\hat{\upsilon}_{1c}}{dx} - \hat{\varphi}_1\right), \quad (21)$$

$$\hat{\bar{M}}_{xx1} = -B_{11}\frac{d\hat{u}_{1c}}{dx} + D_{11}\frac{d\hat{\varphi}_1}{dx}, \qquad \hat{\bar{N}}_{xx2} = A_{11}\frac{d\hat{u}_{2c}}{dx} - B_{11}\frac{d\hat{\varphi}_2}{dx}, \quad (22)$$

$$\hat{\bar{Q}}_{x2} = \kappa A_{12}\left(\frac{d\hat{\upsilon}_{2c}}{dx} - \hat{\varphi}_2\right), \qquad \hat{\bar{M}}_{xx2} = -B_{11}\frac{d\hat{u}_{2c}}{dx} + D_{11}\frac{d\hat{\varphi}_2}{dx}. \quad (23)$$

The results of calculation of the model of acoustic wave propagation dynamics in layers of reinforced composites indicate that only one spectral element is required to represent the tight contact region of the lap joint when there is no gap (e.g., damaged region). The description of the contact region of the double beam is associated with four nodes. Each node has 3 degrees of freedom, for characteristic intervals of the frequency domain, and their solutions in the time domain are obtained using the inverse wavelet transform.

From the solution of the characteristic equation, wave numbers can be determined. The amplitudes of the waves corresponding to each wave number are determined using singular value decomposition. In addition, the solution of the system of partial differential equations must be related to a fixed set of constants, which are determined from the transformed nodal quantities. From the relationship between the displacements and the unknown constants, characteristic operator equations can be written

$$\{\hat{u}^e\} = \{T_1\}\{a\}, \qquad \{\hat{F}^e\} = \{T_2\}\{a\}. \quad (24)$$

The computational model is used to analyze the wave propagation behavior in the frequency and time domains of reinforced composite single lap joints. In the first stage, the frequency domain spectrum and dispersion relations are investigated. The second stage of the analysis operates with the wave propagation results in the time domain. The properties of the laminated composite layers are as follows: E_1 = 144.48 GPa, E_2 = E_3 = 9.63 GPa, G_{23} = G_{13} = G_{12} = 4.128 GPa, m_{23} = 0.3, m_{13} = m_{12} = 0.02 and q = 1389 kg·m^3. The characteristic values of Young's modulus and Poisson's ratio are 4.24 GPa and 0.45, respectively. The shear correction factor was taken to be 5/6. The Daubechies wavelet function has a scaling order of $N = 22$.

Table 1 illustrates the spectral relationships for laminar layers of reinforced composite (ν is the frequency, kHz; k is the wavenumber, m^{-1}).

Table 1 - Spectral relations for reinforced composite

ν	k		
	flexural	shear	modes with i > 3
50	172	2	–
100	263	76	–
150	501	112	0.5
200	792	165	3
250	1004	210	84
300	1217	273	91
350	1408	321	276
400	1572	384	325

Summary and conclusions.

The characteristic systems of equations for the considered model allowed us to obtain the spectral relations of a double reinforced beam with the stacking sequence [010] for each fixed layer. This relation can be related to the solution of the characteristic equation with the sampling time $\Delta t = 1 \cdot 10^{-7}$ s: There are three modes with zero wavenumber values at zero frequency ($\gamma = 0$), which means that none of the three will propagate at this frequency. These fundamental modes are direct analogues of the axial, flexural and shear modes for free-standing beams. The axial and flexural wavenumbers become real values as the frequency increases, while the shear mode does so after a certain cutoff frequency. Three imaginary modes, which start with complex wave numbers at zero frequency $\gamma = 0$ and remain complex up to certain cutoff frequencies. The higher-order modes present in the two-beam system have real numerical values.

The model based on the first order shear deformation assumption gives accurate results at relatively high frequencies. Compactly supported Daubechies scaling functions were used to transform the governing partial differential equations into ordinary differential equations. Complex valued solutions of the governing equation system were obtained using the spectral finite element method, where the dynamic stiffness matrix relates the nodal forces and displacements in the transformed local region of the composite. The complex values of the higher order modes indicated that these modes did not participate in the propagation at low frequencies. It was found that for a certain cutoff frequency, these higher order modes started to propagate, thereby complicating the wave propagation behavior.

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