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METHOD FOR SIDE-LOBES SUPPRESSION IN THE CROSS-CORRELATIONS OF PSEUDO RANDOM SIGNALS WITH A TAILORED REFERENCE

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Abstract. *The paper proposes a new method for estimate of cross-correlations of a periodic m-sequence of maximum length and a tailored reference signal, which allows perfect suppression of its side-lobes along the time delay axis. At the same time, the range resolution and the signal-to-noise ratio at the correlator output did not deteriorate compared to the result of the standard estimate of the autocorrelation function of pseudo random signals.*

Keywords*: Pseudo-Random Sequences, Cross-Correlation Function, Side-Lobes, Tailored Reference*

1. Introduction.

One of the promising class of signals for modern radar are so called pseudo-random signals, the most attractive case of which are the known as $\boldsymbol{m}$-sequences of maximum length, which herein after we will call as *Pseudo-Random Sequences* (PRS) [1-5]. These signals draw researchers' attention due to the extreme simplicity of their generation and processing along with their nice correlation characteristics. Having the above qualities, they serves as basic signals for obtaining new sequences with enhanced performance [6 - 11]. However, to obtain a very low level of side lobes in time delay axis, one have to use $\boldsymbol{m}$-sequences of very large length, since the side lobe level for a periodic PRS is proportional to $1/\boldsymbol{N}$, where $\boldsymbol{N}$ is the number of PRS elements [9 - 11]. Additional suppression of the side lobes may be achieved applying various methods of weighting/windowing, which, in particular, can be performed by the methods of cross-correlation processing [9].

In the paper, the authors propose a novel method for PRS processing that completely suppresses time-delay side-lobes without losses in the Signal-to-Noise Ratio (SNR) at the correlator output.

2. PRS properties and tailored reference signals

Consider cross-correlation of the PRS signal $\boldsymbol{P(t)}$ with a tailored reference signal $\boldsymbol{X(t-\tau)}$, which is the sum of its delayed copy $\boldsymbol{P(t-\tau)}$ and some signal $\boldsymbol{x(t-\tau)}$:

$$\boldsymbol{X(t-\tau) = P(t-\tau) + x(t-\tau)} \quad (1)$$

where τ is the PRS time delay.

Additional signal $x(t-\tau)$ is to be found/defined from the condition of having zero side-lobes of the cross-correlation function:

$$K(\tau) = \overline{P(t)\cdot X(t-\tau)} = \overline{P(t)\cdot[P(t-\tau)+x(t-\tau)]}$$
$$= \overline{P(t)\cdot P(t-\tau)} + \overline{P(t)\cdot x(t-\tau)} \quad (2)$$

where the upper bar means time integration over the PRS period:

$$T = \tau_p N\,,$$

where τ_p is the duration of an PRS elementary pulse; $N = (2^k - 1)$ is the number of elementary pulses in the PRS period, and k is a positive integer $k \geq 2$.

It is well-known, that PRS auto-correlation function equals:

$$\overline{P(t)\cdot P(t-\tau)} = \begin{cases} N\,; & \tau = 0 \\ -1; & \tau_p < \tau \leq T \end{cases} \quad (3a)$$

while its average value equals:

$$\overline{P(t)} = -1\,. \quad (3b)$$

For the time-delays τ from the interval $\tau_p < \tau \leq T$ the auto-correlation function Eq.(3a) has the constant side-lobe level equals to (**-1**). If the second term in Eq.(2) will be equal to **+1** then the side-lobes in Eq.(2) will be cancelled. So from the equation

$$\overline{P(t)\cdot x(t-\tau)} = +1$$

and Eq.(3b) it follows that $x(t-\tau) \equiv -1$ for $\tau_p < \tau \leq T$.

Let's normalize the tailored reference signal making its module equal to **1**:

$$Y(t-\tau) \equiv X(t-\tau)/2 = P(t-\tau)/2 + x(t-\tau)/2 = \frac{[P(t-\tau)-1]}{2} \quad (4)$$

Substituting Eq.(4) into Eq.(2) and using Eq.(3) one may readily obtain:

$$K(\tau) = \overline{P(t)\cdot Y(t-\tau)} = \begin{cases} \frac{N+1}{2}\,; & \tau = 0 \\ 0\,; & \tau_p < \tau \leq T \end{cases} \quad (5)$$

It is seen that the ***cross-correlation*** function in Eq.(5) has zero side-lobe level out of the zone of high correlation, i.e. in the τ delays interval $\tau_p < \tau \leq T$.

Note that autocorrelation functions of both the PRS signal $P(t)$ and the tailored reference signal $Y(t-\tau)$, Eq.(4), have a ***non-zero*** level of their side-lobes. The obtained result is valid for any signals having the property described by Eq. (3).

3. Implementation of the zero side-lobes cross-correlation for PRS signals

Simplified block diagram of the suggested correlation receiver for the case of known phase of the received signal is shown in Figure 1.

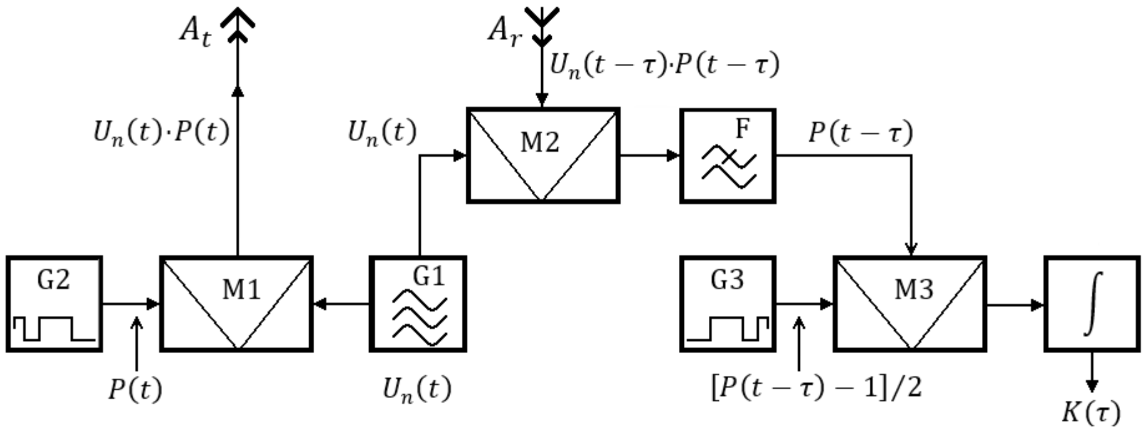

Figure 1. Simplified block diagram of the suggested correlation receiver: *A_t and A_r are transmit and receive antennas, respectively; **G1** is generator of carrier frequency signal; **G2** is generator of baseband PRS for transmit signal formation; **G3** is generator of baseband tailored reference signal; **M1, M2, M3** are mixers in the units for generation of **RF** transmit signal, **Rx** signal frequency down conversion and for cross-correlation estimate, respectively; **F** is low pass filter and $\int$ is integrator.*

Figure 2 shows generated signals to explain the suggested method for particular case of *N=7*. Figure 2.a shows the bipolar baseband PRS signal that was used to generate ***Tx*** phase-keying signal (Figure 2.b). ***RF*** signal $U_n(t)$ from the generator **G1** is multiplied by the unipolar tailored reference signal (generator's **G3** output) in the modulator **M1**, delayed for a time τ with respect to the ***Tx*** PSP signal.

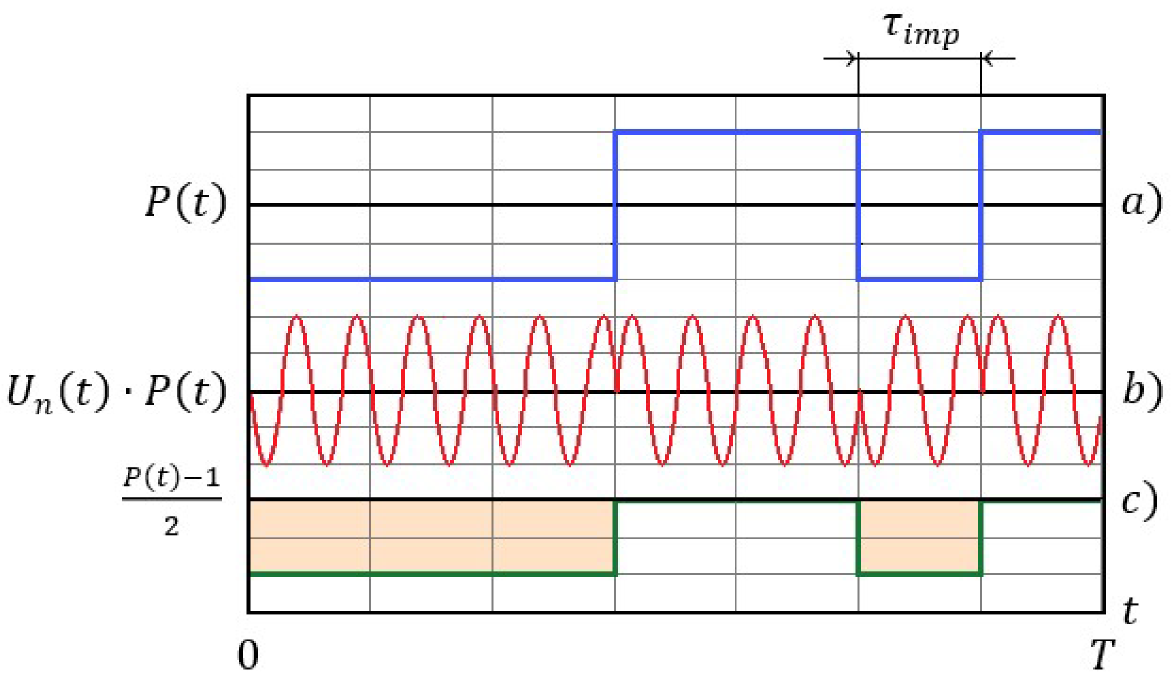

Figure 2. Signals generated in the suggested correlation receiver: *a) baseband bipolar PRS signal, $P(t)$; b) **Tx RF** phase-keying signal; $U_n(t)P(t)$; c) baseband unipolar normalized tailored reference signal: $\frac{[P(t-\tau)-1]}{2}$ (for $\tau = 0$)*

Note that the number of non-zero pulses of the amplitude-manipulated PRS in the interval $\boldsymbol{T}$ (Fig. 2 c)) is always even for a PRS of any length, that is, for any N, which is always odd, since $\boldsymbol{N = 2^k - 1}$. It is this fact that allows us to obtain zero side lobes along the $\boldsymbol{\tau}$ axis for the considered cross-correlation function **K(τ).** This signal and cross-correlations are shown in Figure 3 and Figure 4, respectively. The voltage diagrams at the integrator input are shown in Figure 3 for different values of the delay $\boldsymbol{\tau}$. For $\boldsymbol{\tau = 0}$, the signal at the integrator input is a sequence of pulses of only positive polarity. Their number in the interval $\boldsymbol{T}$ is even and equal to $\boldsymbol{(N+1)/2}$, and the average value is equal also to $\boldsymbol{(N+1)/2}$.

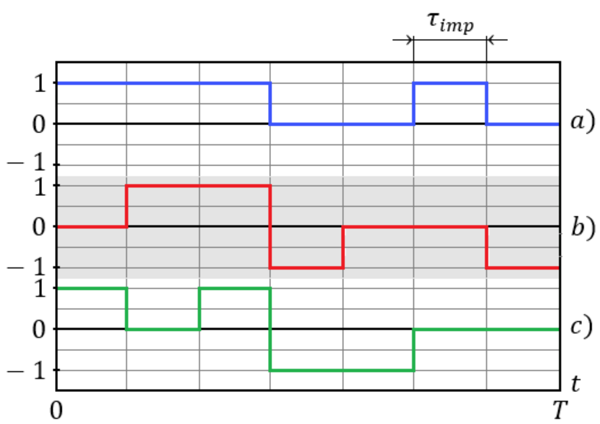

Figure 3. Signal at the integrator input: a) $\boldsymbol{\tau = 0}$; b) $\boldsymbol{\tau = \tau_p}$; c) $\boldsymbol{\tau = 2\tau_p}$.

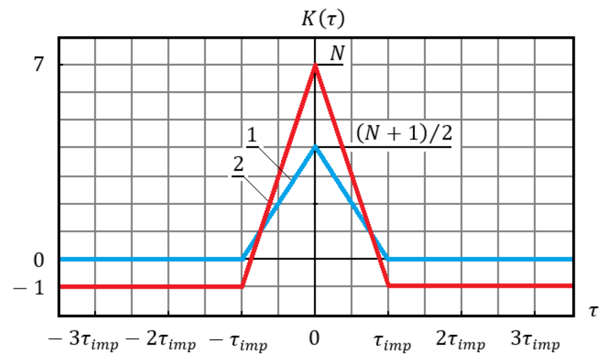

Figure 4. Cross-correlation function $\boldsymbol{K(\tau)}$: graph 1 (blue) is for signals $\boldsymbol{P(t)}$ and $\boldsymbol{[P(t)-1]/2}$; graph 2 (red) is autocorrelation for signal $\boldsymbol{P(t)}$ **.**

At $\boldsymbol{\tau = \tau_p}$, $\boldsymbol{\tau = 2\tau_p}$ and so on, the signal at the integrator input is a sequence of bipolar pulses. Moreover, the number of positive and negative pulses within the interval $\boldsymbol{T}$ is always equal to each other and is $\boldsymbol{(N+1)/4}$, and their average value on this interval is always equal to 0 for $\boldsymbol{\tau_p < \tau \le T}$, which agrees with expression (5).

4. Signal-to-Noise Ratio

Figure 4 shows the cross-correlation function $\boldsymbol{K(\tau)}$ of the signals $\boldsymbol{P(t)}$ and $\boldsymbol{[P(t)-1]/2}$ (curve 1) and, for comparison, the autocorrelation function of the signal **P(t)** (curve 2). As it is seen from Figure 4 and Eq.(5), the cross-correlation function $\boldsymbol{K(\tau)}$ has a smaller maximum value (equal to $\boldsymbol{(N+1)/2}$) than that of the autocorrelation function for the PRS signal $\boldsymbol{P(t)}$. This is a consequence of the fact that the proposed processing uses the energy of not all, but only that of $\boldsymbol{(N+1)/2}$ PRS pulses emitted by the transmitter. At the same time, the resolution along the delay axis $\boldsymbol{\tau}$ and the signal repetition period remain the same and, most importantly, there are no side lobes along the same axis. It is seen from Figure 3 that for large **N,** half of the energy of the received signal is lost, as stated in [11]. However, due to the properties of the tailored reference signal $\boldsymbol{Y}(\boldsymbol{t})$, (4), in the proposed algorithm for processing the received PRS signal (Figure 1), zeroing of the signal at the output of modulator **M3** is implemented in the corresponding time intervals. In essence, this allows gating the noise of the receiver input amplifier. Therefore, the proposed processing scheme does not degrade SNR at the integrator output compared to the traditional PRS signals processing. Indeed, according to traditional processing, the ratio $\boldsymbol{q_1}$ of the signal energy over the time interval $\boldsymbol{T}$ (the PRS period) to the noise energy is:

$$q_1 = \frac{E_{s1}}{E_{n1}} = \frac{A^2 \tau_p N}{D \tau_p N} = \frac{A^2}{D}, \tag{6}$$

where $\boldsymbol{E_{s1}}$ is the signal energy, $\boldsymbol{E_{n1}}$ is the noise energy, $\boldsymbol{A}$ is the effective amplitude of the elementary pulse PSP signal at the mixer output, $\boldsymbol{D}$ is the noise variance at the mixer output. For the proposed processing circuit (Figure 1), which has a gating cascade **M3**, the SNR $\boldsymbol{q_2}$ at the output of this cascade can be determined as follows.

The signal energy and noise energy passed to the integrator input, taking gating into account, are respectively equal to:

$$E_{s2} = A^2 \tau_p \frac{N+1}{2} \tag{7}$$

$$E_{n2} = D \tau_p \frac{N+1}{2} \tag{8}$$

From Eq.(7) we see that the signal energy $\boldsymbol{E_{s2}}$ is less than $\boldsymbol{E_{s1}}$, but the noise energy $\boldsymbol{E_{n2}}$ arriving at the integrator input is also less than $\boldsymbol{E_{n1}}$ (due to the operation of the strobe cascade). As a result, the SNR at the integrator input will remain the same:

$$q_2 = \frac{A^2 \tau_p \frac{N+1}{2}}{D \tau_p \frac{N+1}{2}} = \frac{A^2}{D}. \tag{9}$$

Comparing Eq.(6) and Eq.(9), we come to the conclusion that the proposed processing method for PRS signals has no loss in SNR compared to SNR obtained with the traditional PRS signals processing.

In Figure 5, as an example, the normalized per unit cross-sections of the ambiguity functions along the delay axis $\xi = \boldsymbol{\tau}$ are shown, constructed according to Eq.(16) for sequences with the number of elements **N=63** and **N=15**, curves **2** and **3**, respectively.

In the same figure, for comparison, the normalized cross-section of the ambiguity function for PRSs obtained with conventional processing for the same number of elementary pulses in the sequences is shown as curves **1** and **4** for **N=63** and **N=15**, respectively. The Figure 5 clearly shows that in the latter case the sidelobe level is proportional to $(-\mathbf{1/N})$, while for the proposed processing the side-lobe level is equal to zero for all time delays (curves **2** and **3**).

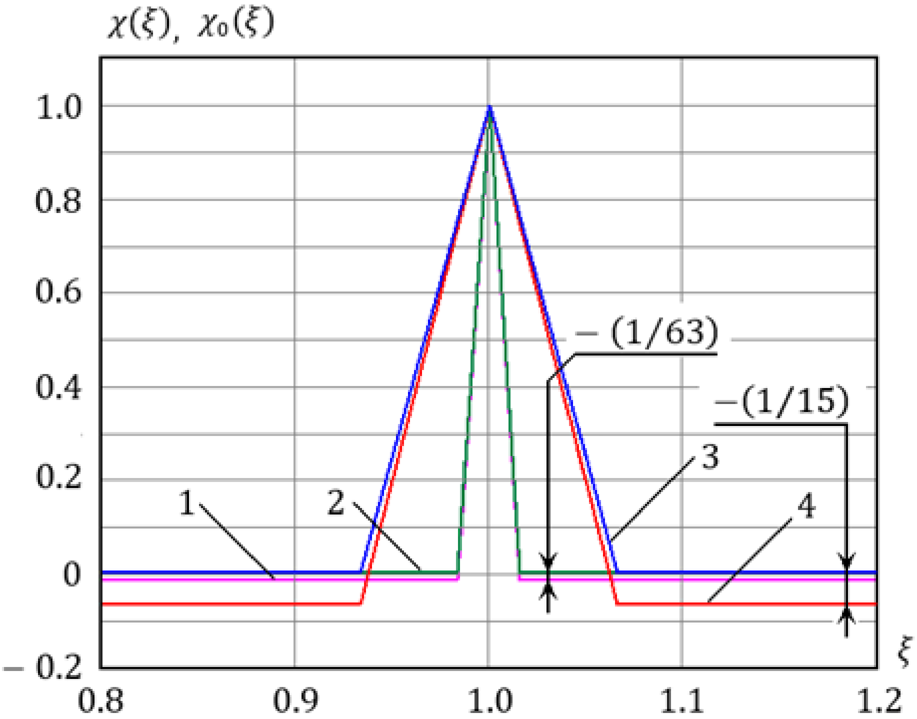

Figure 5. Normalized cross-sections of ambiguity functions for PRS *with **N=63** and **N=15,** obtained with both conventional PRSs signal processing (curves **1** and **4,** respectively) and proposed PRS signal processing (curves **2** and **3**). The side-lobe level for conventional processing of PRS signals with **N=63** and **N=15** equal to **(-1/63)** and **(-1/15),** respectively.*

Conclusions

A method for processing the PRS signal in the form of a periodic **m**-sequence of maximum length is proposed and implemented, which allows one to get rid of the side lobes of the correlation function along the time delay **τ** axis. Moreover, neither the resolution along this axis nor the SNR at the correlator output are worsened in comparison with the classical PRS processing. In particular, this processing method will be very useful when long PRS signals cannot be used in radars, for example due to high target speeds, while the requirements for time-delay (range) side-lobes level are very high. In addition, the proposed method may be easily adapted for a digital implementation and, hence, is very easily amenable to a high degree of integration.

References

1. Skolnik, Merrill I. Introduction to Radar Systems. McGraw-Hill, 2001.

2. Richards, Mark A. Fundamentals of Radar Signal Processing. McGraw-Hill, 2014.

3. John G. Proakis, Dimitris K Manolakis. Digital Signal Processing: Principles, Algorithms, and Applications. 4th Edition. Prentice Hall. P. 1087. 2006

4. Alan V. Oppenheim, Ronald W. Schafer. Discrete-Time Signal Processing. 3rd Edition. Publishing house: Prentice Hall. P. 1108. 2009

5. Costas, J.P. ‘A study of a class of detection waveforms having nearly ideal Range-Doppler ambiguity properties’, Proc. IEEE, 1983, 72, (8), pp. 996–1009

6. Trachtman A.M., Trachtman V.A. Fundamentals of the theory of discrete signals on finite intervals. - M., Sov. Radio, 1975, 208 p.

7. Varakin L.E. Theory of Complex signals. - M., Sov. radio, 1970, 375 p.

8. Shirman Ya. D. Theoretical basis of radiolocation. -M.: Sov. radio, 1970, 560p.

9. Mark A. Richards. Fundamentals of Radar Signal Processing. 2nd ed. Publishing house: McGraw-Hill Education. 2014, 656 p.

10. R.N.Mutagi, ‘Pseudo Noise Sequences for Engineers’, Electronics and Communication Engineering Journal, UK, Vol.8, No.2, April 1996

11. F. Jessie MacWilliams, Neil J. A. Sloane, Pseudo-Random Sequences and Arrays, IEEE, p. 1715-1729.