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BOUNDARY CONDITIONS OF LAMINATED COMPOSITES VIBRATION

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Abstract. *Vibration of laminated composite cylindrical shells can lead to undesirable resonance effects and even failure of mechanical system components. The aim of this study is to develop a discrete wavelet analysis of free vibrations of cylindrical shells under various boundary conditions. The study uses the basic concepts of the theory of mechanical shells. The relationship between mechanical stresses and shears is described by a system of partial differential equations. The partial differential equations are first transformed into a system of ordinal differential equations by separating the variables. The discrediting procedure is applied to the governing equations. Cylindrical shells were modeled from an arbitrary number of orthotropic plates, which were rigidly fastened together. The boundary conditions of the basic types are formulated using discrete wavelet analysis, which ultimately allows to describe a standard linear eigenvalue problem. This study extends the application of discrete wavelet analysis to the description of free vibrations of cylindrical shells. It modifies the traditional model by taking into account the influence of boundary conditions, lamination schemes, and elastic moduli on the natural frequencies of vibrations. The characteristics of free vibration modes of cylindrical shells predicted by this model can be used for laminated composite samples located on a non-uniform elastic foundation. In this case, data on the localized increase in the stiffness of the composite material can be used to calculate the intervals of vibration stability. A numerical model based on the discrete wavelet transform was applied to the analysis of free vibrations of composite laminated cylindrical shells under different boundary conditions. Calculations using this model were characterized by fast convergence and high accuracy. The effects of such essential factors as boundary conditions, the structure of laminated composites, and their effective stiffness moduli on the natural frequencies of free vibrations of the shells were analyzed.*

Key words: *laminated composites, discrete wavelet analysis, cylindrical shells, free vibrations, modulus of rigidity.*

Introduction.

Numerical analysis of the characteristics of beams, plates and shells of revolution in a static or dynamic state, resting on an elastic foundation, is usually based on approximate models of the elastic foundation [1]. The reaction of the foundation is described by differential operators acting on the deflections of elastic bodies. A large number of studies are devoted to the analysis of the influence of an elastic foundation on the linear or nonlinear vibrations of circular cylindrical shells [2]. In particular, natural frequencies of oscillations were obtained for simply supported cylindrical shells [3, 4]. Numerical values of characteristic coefficients of natural oscillations vary in a wide range for frequencies corresponding to radial, longitudinal and torsional modes [5, 6].

The three-dimensional case of free vibrations of thick-walled cylindrical shells immersed in a two-parameter elastic medium can also be characterized by a limited number of modes with different boundary conditions and with different combinations of characteristic coefficients. It should be noted that such properties of the elastic foundation as inertia also affect the natural vibrations of three-layer shells. In particular, the presence of an elastic medium significantly increases the frequencies of radial vibrations of three-layer shells with a thick filler [7, 8]. The numerical values of natural frequencies, as well as the form factors of vibrations, nonlinearly depend on the variable thickness of cylindrical isotropic and orthotropic shells. Experiments indicate an increase in the influence of an elastic foundation with an increase in the ratio of the maximum thickness to the minimum.

Local gradients of mechanical stresses on the surfaces of the functionally graded shell of reinforced composites, split into several layers and immersed in an elastic foundation of the Winkler type, lead to a decrease in the frequencies of free oscillations. Analysis of the spectrum of nonlinear coefficients of the elastic foundation allows localizing the regions of increasing oscillation frequencies on the surface of cylindrical shells of laminar composites [9].

In the case of uniaxial tension, the effect of discretization of the graded layer into a number of homogeneous sublayers occurs, each of which is characterized by its own displacement coefficient. The increase in the rigidity of the elastic medium is the reason that the influence of geometric nonlinearity, material heterogeneity, the number of winding layers and the magnitude of the reinforcement angles on the oscillation frequencies is reduced.

Most of these methods were first applied to isotropic cylindrical shells and then extended to study the dynamic behavior of anisotropic and layered composite shells. However, despite the various methods of analytical and computational analysis of cylindrical shell structures, finding reliable and efficient approaches for the considered structures with different boundary conditions still remains a big problem.

Therefore, the aim of this paper is to introduce the Haar wavelet approach for the analysis of free vibrations of composite layered cylindrical shells. The free oscillation model used the Haar wavelet, which consists of pairs of piecewise constant functions and one of the simplest orthonormal wavelets with a compact support. A limited set of orthonormal wavelets generated from the same parent wavelet form a basis. The elements of the wavelet basis are orthonormal to each other and normalized to unit length. This property allows each wavelet coefficient to be calculated independently of other wavelets.

Materials and results

The Haar wavelet family $h_i(\xi)$ is defined for $\xi \in [0, 1]$. The matrix H of Haar characteristic coefficients based on l is defined as $H(i, l) = h_i(\xi_l)$. The corresponding matrix $P^{(\alpha)}(i, l)$ of integral transformations has dimensions of $2M \times 2M$. Let us consider a model of a composite layered cylindrical shell. In this model, the length, average radius and thickness of the shell are designated as L, R and h, respectively. The main surface of the shell can be considered as the median surface on which the orthogonal coordinate system (x, θ and z) is fixed. The x, θ and z axes are taken in the axial, circumferential and radial directions, respectively. The displacements of the shell in the x, θ and z directions are designated as u, υ and w.

The deformation at the mean surface (ε_0) and the change in curvature (χ_0) during deformation with transposition operator T are functions of the displacement

$$\varepsilon_0 = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^T, \qquad \chi_0 = [\chi_1, \chi_2, \chi_3]^T \tag{1}$$

$$\varepsilon_1 = \frac{\partial u}{\partial x}, \quad \varepsilon_2 = \frac{\partial \upsilon}{R\partial \theta}, \quad \varepsilon_3 = \frac{\partial \upsilon}{\partial x} \tag{2}$$

$$\chi_1 = -\frac{\partial^2 w}{\partial x^2}, \quad \chi_2 = \frac{\partial \upsilon}{R^2 \partial \theta}, \quad \chi_3 = -\frac{\partial^2 w}{R\partial x \partial \theta}. \tag{3}$$

The governing equations for vibrations can be expressed as in the following form of stiffness matrixes A, B and differential operators $L_{ij} = L_{ij}(A, B, x, \theta)$

$$L_{11}u + L_{12}\upsilon + L_{13}w = (A,B)\frac{\partial^2 u}{\partial t^2}, \tag{4}$$

$$L_{21}u + L_{22}\upsilon + L_{23}w = (A,B)\frac{\partial^2 \upsilon}{\partial t^2}, \tag{5}$$

$$L_{31}u + L_{32}\upsilon + L_{33}w = (A,B)\frac{\partial^2 w}{\partial t^2}. \tag{6}$$

This model considers boundary conditions of the following types: *BC1* (clamped edge), *BC2* (simply supported edge) and *BC3* (free edge). They are defined as follows:

$$BC1: U = 0,\ V = 0,\ W = 0,\ \frac{dW}{dx} = 0, \tag{7}$$

$$BC2: V = 0, W = 0,\ N_x = 0, M_x = 0, \tag{8}$$

$$BC3: M_x = N_x + \frac{M_{x0}}{R} = Qx + \frac{M_{x0}}{R}, \tag{9}$$

The Haar wavelet discretization method was used to discretize the derivatives in the control equations in terms of displacements and boundary conditions [10]. A necessary condition for solving the finite field problem is the transformation of the displacement field into a unit interval. Transformation of a series of wavelets leads to a discrete system of algebraic equations with respect to one normalized variable ξ. The higher-order derivatives of these solutions with respect to the axial coordinate can be expanded in terms of completed Haar wavelets [11]. Thus the following non-dimensional variable is introduced

$$\xi = x / L. \tag{10}$$

Using this parameter, one can obtain a discrete system of algebraic equations with respect to one normalized variable n. The higher-order derivatives of these solutions with respect to the axial coordinate can be expanded using a series of complete Haar wavelets. Therefore, it is assumed that the solutions can be expressed as ($n = 2M$)

$$\frac{d^2 U(\xi)}{d\xi^2} = \sum_{i=1}^{n} a_i h_i(\xi) \tag{11}$$

$$\frac{d^2 V(\xi)}{d\xi^2} = \sum_{i=1}^{n} b_i h_i(\xi) \tag{12}$$

$$\frac{d^2W(\xi)}{d\xi^2}=\sum_{i=1}^{n}c_i h_i(\xi), \tag{13}$$

where a_i, b_i, and c_i are the unknown wavelet coefficients and h_i (ξ) is the Haar wavelet series.

In this case, the displacement amplitudes can be represented in the following form

$$\frac{dU(\xi)}{d\xi}=\sum_{i=1}^{n}a_i p_{1,i}(\xi)+\frac{dU(0)}{d\xi} \tag{14}$$

$$\frac{dV(\xi)}{d\xi}=\sum_{i=1}^{n}b_i p_{1,i}(\xi)+\frac{dV(0)}{d\xi} \tag{15}$$

$$\frac{dW(\xi)}{d\xi}=\sum_{i=1}^{n}a_i p_{3,i}(\xi)+\frac{1}{2}\xi^2\frac{d^3W(0)}{d\xi^3}+\xi\frac{d^2W(0)}{d\xi^2}+\frac{dW(0)}{d\xi}, \tag{16}$$

where in the case $i > 1$ and $\xi^{(2)} < \xi < \xi^{(3)}$ we get

$$p_{n,i}(\xi)=(n!)^{-1}\left[\left(\xi-\xi^{(1)}\right)^n-2\left(\xi-\xi^{(2)}\right)^n\right]. \tag{17}$$

For solving boundary value problems, the values $P_{n,i}$ (0) and $P_{n,i}$ (1) should be calculated in order to satisfy the boundary conditions.

The evaluation of the system of equations at the collocation points can be written in matrix form as

$$U=P_1\begin{bmatrix}a\\d\end{bmatrix},\ V=P_2\begin{bmatrix}b\\e\end{bmatrix},\ W=P_3\begin{bmatrix}c\\f\end{bmatrix}. \tag{18}$$

In the matrix equation, the appearance of eight integration constants allows adding eight additional equations. Using boundary conditions, additional equations can be obtained. The current wavelet transform technique offers an exact solution for cylindrical shells with various boundary conditions. It should be noted that all types of classical boundary conditions can be easily implemented for the type of laminar composites under consideration.

Using boundary conditions, additional equations can be obtained. The current method offers an exact solution for cylindrical shells with different boundary conditions. All classical boundary conditions can be easily calculated. The governing equations and the corresponding boundary condition equations were discretized using wavelet transforms. From the above procedures, a general relationship was obtained for the displacement vector $X = [U, V, W]$, displacement matrix K, and local masses matrix M of laminated composites cylindrical shells

$$\left(K-\omega^2 M\right)X=0. \tag{19}$$

The following values of physical quantities were used in the calculation part of the model: $R = 1.2$ m; $L/R = 4.5$; $h/R = 0.02$; $E_2 = 12$ GPa; $E_1/E_2 = k$, $k \in [2.5 - 20]$; $G_{12} = 5.1$ GPa (shear modulus); $\rho = 1650$ kg/m^3. The following ratio was used as the reference frequency $\Omega = \omega R(\rho_0/E_2)^{0.5}$. The base frequency was calculated for both the three main boundary conditions *BC1*, *BC2* and *BC3* and for the intermediate boundary condition *BC1-BC2*.

The calculation results showed that the frequency parameter of the shell with the boundary condition *BC* 1 is higher than *BC* 2 when the circumferential wave number n is fixed at a constant value $n = 1$. The reason for this is that the boundary conditions have a noticeable effect on the shell's natural frequencies. However, it should be noted that the lowest fundamental frequency parameter occurs for the boundary condition *BC* 3 of the cylindrical shell. In other words, the fundamental frequency of composite cylindrical shells is not necessarily related to the lowest circumferential wave number.

An additional numerical analysis was performed to investigate the effect of complex lamination patterns on the frequencies of laminated cylindrical shells. The frequency parameters were determined for cross-laminated cylindrical shells. These shells had a small thickness ratio ($h/R = 0.02$) and a moderate length ($L/R = 4$). In addition, for simplicity, it was assumed that all layers had the same thickness.

Thus, it can be concluded that the developed method accurately predicts the natural frequencies of laminated cylindrical shells with different lamination schemes. Analysis of the calculation results allows us to state that the frequency parameters for the [90°/0°/90°] lamination are greater than for [0/0/0], and the frequency parameters for the [90°/90°/90°] and [0°/90°/0°] cases are between them.

The frequency parameters are classified not according to their wave number value, but according to their order in the direction of the larger radius of curvature. The values of characteristic frequency Ω for the vibrations of cylindrical shells of laminated composites depending on the wave number *n* are shown in Table 1.

Table 1 - Frequency parameter Ω for different boundary conditions

n	Boundary conditions			
	BC1	*BC2*	*BC3*	*BC1-BC2*
1	0.2012	0.3052	0.1053	0.2317
2	0.2578	0.3667	0.1537	0.2828
3	0.3001	0.4235	0.2074	0.3235
4	0.3665	0.4578	0.2519	0.3821
5	0.4019	0.5104	0.3039	0.4378
6	0.4631	0.574	0.3541	0.4758
7	0.5197	0.6287	0.4106	0.5331
8	0.5690	0.6729	0.4583	0.5825
9	0.6107	0.7342	0.5195	0.6308
10	0.6655	0.7815	0.5528	0.6766
11	0.7030	0.8382	0.6180	0.7253
12	0.7599	0.8861	0.6504	0.7842
13	0.8046	0.9411	0.7011	0.8306
14	0.8611	0.9775	0.7544	0.8808

The calculation results indicate that the frequency parameters for laminated composites with a large number of shells are significantly larger than the corresponding parameters for composites with two or three shells. This property can be explained by the fact that the orthotropic material is stiffer in the axial direction than in any other direction. The stiffness in the presence of a large number of shells can be maximum, and thus the frequency value is also the highest.

Summary and conclusions.

A computational model based on the Haar wavelet discretization method was applied to the analysis of free vibrations of composite laminated cylindrical shells. The vibrations of a laminated composite sample occurred under different boundary conditions. The characteristics of mechanical vibrations were calculated based on the classical shell theory. The discretization method of the control equations and the corresponding boundary conditions was implemented using discrete wavelet transforms. It was found that boundary conditions, length-to-radius ratios, lamination schemes, and elastic moduli ratios affect the natural frequency parameters of cylindrical shells made of laminated composites. The discrete wavelet analysis technique can also be used to describe vibrations of thick composite laminated and functionally graded shells.

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