http://www.moderntechno.de/index.php/meit/article/view/meit41-02-032
DOI: 10.30890/2567-5273.2025-41-02-032

UDC 699.88

PROPAGATION OF SYMMETRIC AND ANTISYMMETRIC MODES IN MULTILAYER COMPOSITE PLATES

Pysarenko A.M.
c.ph.-m.s., as.prof.
ORCID: 0000-0001-5938-4107
Odessa State Academy of Civil Engineering and Architecture,
Odessa, Didrihsona, 4, 65029

Abstract. *In this model, a single layer of a homogeneous isotropic elastic composite plate is described by dispersion relations of the following type symmetric and antisymmetric modes of Lamb waves. The composite plate is modeled as a homogeneous transversely isotropic material with the x3-axis as the axis of symmetry. An analytical plane strain model can be considered without loss of generality and this will be sufficient to determine the propagation characteristics of directed waves. For the case of a transversely isotropic composite material, the model problem can be divided into the motions of symmetric and antisymmetric modes of Lamb wave packets. The standard laminar composite flaw detection technique includes a model delamination object in the form of a discontinuity with reduced bending stiffness in the delamination region. The reduction in bending stiffness in the delamination region can be explained by the separation of the laminate in this region into upper and lower sublayers, in which the waveguide is divided into two separate subwaveguides. The presence of a discontinuity can cause both reflected and transmitted waves from the delamination. However, in composite laminates, the scattering of Lamb waves at delaminations is a rather complex phenomenon. In this regard, it is necessary to evaluate the accuracy of the equivalent isotropic model in predicting the scattering characteristics of the Lamb A0 wave at delaminations in composite laminates. The method involves the analysis of the characteristics of scattered Lamb waves A0, which were obtained from a limited number of monitoring points by calculating the difference between the signal from the undamaged plate and the signal from the damaged plate.*

Key words: *scattering, delamination, composite laminates, Lamb waves, transversely isotropic material, bending stiffness.*

Introduction.

The utilization of composite materials, particularly in the form of multilayer plates, has become ubiquitous across various advanced engineering sectors, including aerospace, automotive, and civil infrastructure, driven by their exceptional strength-to-weight ratio, high stiffness, and customizable mechanical properties. Ensuring the structural integrity and long-term reliability of these complex structures is paramount, necessitating robust and highly sensitive non-destructive evaluation techniques. Among the most effective methods for inspecting large-area, thin-walled composite structures is the use of guided waves, specifically Lamb waves, which can propagate over significant distances while remaining sensitive to various forms of damage such as delamination, cracks, and porosity. Lamb waves are elastic waves that travel within a solid plate or shell and are governed by the plate boundaries, exhibiting distinct wave modes that are dispersive, meaning their speed depends on the frequency-thickness product. This characteristic dispersion, while complicating analysis, also provides a rich source of information for damage detection. The analysis of Lamb wave propagation in single-layer isotropic plates is a well-established field, characterized by two fundamental families of modes: symmetric modes, often denoted S0, which involve predominantly in-plane motion and symmetric displacement across the mid-plane, and antisymmetric modes, denoted A0, which involve predominantly out-of-plane motion and antisymmetric displacement across the mid-plane [1]. The S0 and A0 modes are particularly important in non-destructive testing due to their low-frequency nature and ability to travel long distances with minimal attenuation. However, when moving from a single isotropic layer to a multilayer composite plate, the complexity of wave propagation increases significantly due to several factors. Firstly, the material properties within each layer are typically anisotropic, often exhibiting orthotropy or transverse isotropy, a direct result of the fiber reinforcement direction. Secondly, the interaction between layers through interfaces, which may involve perfect bonding or slight imperfections, introduces further complexity into the boundary conditions. Thirdly, the presence of multiple layers with potentially varying thicknesses and material orientations leads to a much larger number of possible wave modes, as the total number of modes in a multilayer structure is a superposition of the modes that could exist in each individual layer, coupled through the interface boundary conditions. Understanding the characteristics of these symmetric and antisymmetric modes in multilayer composite plates is thus a foundational requirement for successful guided wave inspection. The study of wave propagation in these complex media relies on sophisticated theoretical frameworks, primarily employing effective medium theories or, more commonly and accurately, the global matrix method, the transfer matrix method, or the stiffness matrix method, all of which solve the Christoffel equation subject to the boundary conditions at the free surfaces and the continuity conditions at the layer interfaces. These methods yield the dispersion relations such as equations linking wave frequency, phase velocity, and group velocity that are essential for predicting wave behavior and for interpreting experimental results. The symmetric and antisymmetric classification, which is straightforward for a single isotropic layer, must be carefully considered for multilayer composites [2]. If the plate as a whole is structurally and materially symmetric with respect to its mid-plane (i.e., a laminate with a symmetric stacking sequence and symmetric properties), the wave modes can still be rigorously classified as symmetric or antisymmetric with respect to the mid-plane displacement and stress profiles. However, in the case of unsymmetric laminates, where the stacking sequence or material properties are not mirrored across the mid-plane, the wave motion generally becomes coupled, meaning the concepts of purely symmetric and purely antisymmetric modes no longer strictly apply. Instead, the modes exhibit mixed characteristics, though they are often still loosely referred to as quasi-symmetric or quasi-antisymmetric based on the dominant displacement component. The practical application of guided waves in composite inspection is critically dependent on accurately predicting the behavior of these modes. For instance, the A0 mode, characterized by its low phase velocity at low frequencies, is often highly sensitive to delamination and out-of-plane defects due to its significant out-of-plane displacement component, while the S0 mode is often more sensitive to in-plane defects like fiber breaks or matrix cracks. The ability to select the optimal excitation frequency and mode type is crucial for maximizing defect detection probability and minimizing inspection time. Furthermore, the strong dispersion exhibited by the higher-order modes in multilayer composites means that a wave pulse, composed of multiple frequencies, will spread out as it propagates, which can complicate signal analysis and time-of-flight measurements, necessitating the use of advanced signal processing techniques such as the wavelet transform or two-dimensional Fourier transform to isolate individual modes. Research in this field is continually advancing, focusing on developing more efficient and accurate numerical models, understanding the effects of material damping and viscoelasticity on propagation, and, most importantly, modeling the complex interaction and scattering of these wave modes with various types of damage that are characteristic of multilayer composite structures. The fundamental theoretical understanding of symmetric and antisymmetric mode propagation in these complex, anisotropic media remains the bedrock upon which all practical non-destructive evaluation strategies for multilayer composite plates are built. The pursuit of an accurate and efficient theoretical framework for predicting these dispersive characteristics is essential for the future development of reliable structural health monitoring systems for these ubiquitous high-performance materials.

Dispersion relations of the following type symmetric and antisymmetric modes

The zero-crossing algorithm assumes the possibility of decomposing the measured signals u_i(t) at different distances into a set of signals with a limited bandwidth u_{ik}(t). Such a procedure becomes possible provided that the signals u_i(t) are filtered using bandpass filters with a narrower bandwidth than the bandwidth of the incident spectrum.

At the next step for each filtered signal $u_{ik}(t)$ the delay times t_{imk} and $t_{(i+1)mk}$ are estimated. In this way, the phase velocities c_{pmk} and frequencies f_{mk} are calculated. Then, the obtained sets (f_{mk}. c_{pmk}) can be represented as segments of the phase velocity dispersion curve.

Each section of the dispersion curve obtained using one of the filters can be reconstructed in a relatively narrow passband. In addition, scanning the filter's central frequency in wide frequency ranges will allow covering a large part of the falling spectrum.

The frequency spectrum of two adjacent signals can be represented as

$$U_i(f) = FT[u_i(t)]. \quad (1)$$

$$U_{i+1}(f) = FT[u_{i+1}(t)]. \quad (2)$$

where $u_i(t)$ is the signal measured at distance $x_i(t)$;

$u_{i+1}(t)$ is the signal measured at distance $x_{i+1}(t)$;

FT is the Fourier transform.

Adjacent bands of the frequency spectra are filtered by k Gaussian bandpass filters with predetermined parameters

$$U_{ik}(f) = U_i(f) \cdot B_k(f). \quad (3)$$

$$U_{(i|+1)k}(f) = U_{i+1}(f) \cdot B_k(f). \qquad (4)$$

where the frequency response of k-th bandpass filter is

$$B_k(f) = \exp\left\{4\ln(0.5) \cdot \Delta B^{-2} \cdot [f - f_L - (k-1)df]^2\right\}. \quad k = 1,2,...,K. \qquad (5)$$

where f_L is the left frequency filter edge;

f_H is the central part of frequency filter edge;

ΔB is the filter bandwidth;

the frequency domain df is

$$df = \frac{f_H - f_L}{K-1}. \qquad (6)$$

The signal reconstruction using the Fourier transform has the following form

$$u_{ik}(t) = FT^{-1}[U_{ik}(f)]. \qquad (7)$$

$$u_{(i\backslash+1)k}(t) = FT^{-1}[U_{(i+1)k}(f)]. \qquad (8)$$

At the next stage of the numerical method. the phase velocity can be estimated according to the following relation

$$c_{pmk} = \frac{x_{i+1} - x_i}{t_{(i+1)mk} - t_{imk}}. \qquad (9)$$

Using the data on the duration of the half-periods of the first signal, it is possible to estimate the equivalent frequencies to which the calculated values of the phase velocity should be assigned

$$f_{imk} = \frac{0.5}{t_{i(m+1)k} - t_{imk}}. \qquad (10)$$

Comparison of the results obtained using the proposed hybrid method and the previous version of the reference zero-crossing method showed that the zero-crossing method recovers the phase velocity dispersion curve in the frequency range of 286–319 kHz. This bandwidth is only 8% of the original signal bandwidth.

Meanwhile, the proposed spectrum decomposition approach allows us to recover the phase velocity dispersion curve in a significantly wider frequency range. This range covers almost the entire bandwidth of the incident Lamb wavelet signal. t should be noted that the frequency ranges in which the dispersion curve is reconstructed depend significantly on the bandwidth of the filters used in the spectrum decomposition approach.

For example, for the 120 kHz filter, 70% coverage of the incident signal bandwidth was achieved. Finally, the best results were achieved with the narrowest filter (40 kHz bandwidth). For this narrowed filter, the reconstructed dispersion curve covers 90% of the original bandwidth.

In summary, it can be argued that narrow filters are more efficient, but this leads to a large number of filters, which generates more computational resources and longer processing times.

Typical B-scan results for the propagation of the A0 Lamb wave mode in a 2 mm thick laminated composite sample are presented in Table 1.

Table 1 - Dispersion dependences for A0 mode of the Lamb wave

A-band		B-band		C-band	
f, kHz	*A′*	*F*, kHz	*A′*	*F*, kHz	*A′*
15.9304	0.0152	157.2193	0.5169	304.7936	0.5203
25.3012	0.0338	158.1551	0.4882	311.3182	0.5458
34.6720	0.1083	162.8342	0.5017	318.7750	0.5085
41.2316	0.2183	170.3209	0.4595	322.5033	0.5864
47.7912	0.2470	176.8717	0.5456	328.0959	0.6661
58.0991	0.2369	185.2941	0.5253	327.1638	0.7322
62.7845	0.3063	187.1658	0.6284	333.6884	0.6898
61.8474	0.4856	195.5882	0.7128	341.1451	0.7254
69.3440	0.6125	199.3316	0.7010	343.9414	0.6644
74.0295	0.4399	199.3316	0.7517	347.6698	0.5458
78.7149	0.3012	203.0749	0.7973	356.9907	0.5746
85.2744	0.4467	209.6257	0.7720	361.6511	0.6085
87.1486	0.5330	213.3690	0.7483	364.4474	0.5339
89.9598	0.6261	213.3690	0.8142	363.5153	0.4864
96.5194	0.5753	217.1123	0.8953	362.5832	0.3814
104.0161	0.7547	218.9840	0.9865	366.3116	0.2712
111.5127	0.5381	233.0214	0.8666	377.4967	0.1780
113.3869	0.6819	237.7005	0.6791	387.7497	0.1051
115.2610	0.7902	246.1230	0.7500	399.8668	0.1322
122.7577	0.9306	248.9305	0.8497	411.9840	0.1797
126.5060	0.8190	254.5455	0.9662	424.1012	0.1102
136.8139	0.7580	263.9037	0.9578	430.6258	0.0678
143.3735	0.7733	266.7112	0.8868	441.8109	0.1186

The dispersion curve data for the relative amplitude are divided into three ranges: A-band (0 - 150 kHz). B-band (150-300 kHz) and C-band (300-500 kHz).

In this model, a single layer of a homogeneous isotropic elastic composite plate is described by dispersion relations of the following type symmetric and antisymmetric modes of Lamb waves

$$\left(2k^2 - k_2^2\right)^2 \cosh(\eta_1 H)\sinh(\eta_2 H) - 4k^2\eta_1\eta_2 \sinh(\eta_1 H)\cosh(\eta_2 H) = 0 . \quad (11)$$

Accordingly, for antisymmetric modes of Lamb waves, the equation can be modified as follows

$$\left(2k^2 - k_2^2\right)^2 \sinh(\eta_1 H)\cosh(\eta_2 H) - 4k^2\eta_1\eta_2 \cosh(\eta_1 H)\sinh(\eta_2 H) = 0, \quad (12)$$

where

$$\eta_j = \sqrt{k^2 - k_j^2}, \quad k_j = \frac{\omega}{c_j}, \quad j = 1,2, \quad (13)$$

$k = \omega/c$ is the angular wavenumber;

H is half the thickness of the composite sample;

c_1 is the P-wave velocity for lamina composite material;

c_2 is the S-wave velocity for lamina composite material.

The working fluid for the model calculation experiment was a laminar composite consisting of several layers with the stacking sequence $[0^0/45^0/0^0/45^0]$s. The composite plate is modeled as a homogeneous transversely isotropic material with the x_3-axis as the axis of symmetry.

Young's modulus E_{22} can be determined from the results of a standard uniaxial tensile test in the x_2 direction. In addition, a stress-strain curve is plotted from the tensile test results. Poisson's ratio ν_{12} in the 1-2 plane is determined from the same test by measuring the strains in both the x_1 and x_2 directions. The shear modulus G_{23} and the elastic constant E_{33} for the composite face sheet are determined from the equations

$$G_{23} = \upsilon_{23}^2 \rho , \quad (14)$$

$$E_{33} = \frac{\rho E_{22}(1-\nu_{12})\upsilon_{11}^2}{E_{22}(1-\nu_{12}) + 2\rho\nu_{13}^2\upsilon_{11}^2}, \quad (15)$$

where υ_{11} is the longitudinal wave velocity;

υ_{23} is the shear wave velocity;

An analytical plane strain model can be considered without loss of generality and this will be sufficient to determine the propagation characteristics of directed waves. For the case of a transversely isotropic composite material, the model problem can be divided into the motions of symmetric and antisymmetric modes of Lamb wave packets.

Summary and conclusions. The structural material under investigation is a laminar composite consisting of several layers with a symmetric stacking sequence of layers. For the purpose of modeling, the composite plate is simplified to an equivalent homogeneous transversely isotropic material, with the x3-axis designated as the axis of symmetry, which is a common and effective simplification for predicting overall guided wave behavior. The theoretical analysis employs an analytical plane strain model, which is sufficient and incurs no loss of generality for accurately determining the propagation characteristics of the guided waves. A key conclusion is that for this transversely isotropic composite material model, the overall wave propagation problem can be effectively and cleanly separated into the motions of distinct symmetric and antisymmetric modes of Lamb wave packets, confirming the utility of this modal decomposition for both theoretical analysis and practical signal interpretation in non-destructive evaluation. The model establishes a robust framework for relating material properties, derived from standard mechanical tests, to the dispersive characteristics of Lamb waves in multilayer composites.

References:

1. Sharma, J. N., & Pal, M. (2004). Propagation of Lamb waves in a transversely isotropic piezothermoelastic plate. Journal of Sound and Vibration, 270(4-5), 587-610. DOI: 10.1016/S0022-460X(03)00093-2

2. Wang, L., & Yuan, F. G. (2007). Group velocity and characteristic wave curves of Lamb waves in composites: Modeling and experiments. Composites science and technology, 67(7-8), 1370-1384. DOI: 10.1016/j.compscitech.2006.09.023

Article sent: 17.10.2025